Day 22 homework - Assigned 3/6 and due 3/27

Starred problems below are extra-credit for undergraduates and required for graduate students.

7. (a) Let $H = \{1, 19\}$ be a subgroup of $U(30)$. Find all the left cosets of H in $U(30)$.

(b) Write out the left cosets of the subgroup $H = \{(1), (1, 2)\}$ of S_3. Write out the right cosets. Is every left coset also a right coset?

8. Let Z be the group of integers under addition and let n be a positive integer. Let $H = \langle n \rangle$ be the cyclic subgroup generated by n. Prove that $aH = bH$ if and only if $a \equiv b \pmod{n}$.

9. $*$ Let G be a finite group and H a subgroup of G. Let S be the set of all left cosets of H in G.

(a) Given an element $g \in G$, define $T_g : S \to S$ to be the function given by $T_g(aH) = gaH$. Show that this function is well-defined. (This means to show that if $aH = bH$, then $gaH = gbH$.

(b) Show that each T_g is a permutation of the set S.

(c) Let $\overline{G} = \{T_g : g \in G\}$ and define $\phi : G \to \overline{G}$ by $\phi(g) = T_g$. Show that ϕ is operation-preserving.

(d) Prove that $\phi : G \to \overline{G}$ is an isomorphism if and only if the only element of G contained in all of the conjugates aHa^{-1} of H is the identity.