Day 3 homework - Assigned 1/17 and due 1/24

Starred problems below are extra-credit for undergraduates.

5. Define the Fibonacci sequence by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Use induction to prove that if $M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then

$$M^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}.$$

6. Let S be the set of 2×2 matrices $M_2(\mathbb{R})$. For $a, b \in S$, define $a \sim b$ if $ab = ba$. Is this an equivalence relation? Why or why not?

7. Let S be the set of positive integers. For $a, b \in S$, define $a \sim b$ if there is a positive integer k so that $ab = k^2$. Is this relation an equivalence relation? Why or why not?

8. Let $S = \{s_1, s_2, \ldots, s_n\}$ and $T = \{t_1, t_2, \ldots, t_n\}$ be two sets that have the same size.

(a) Suppose that $f : S \to T$ is one-to-one. Prove that f is onto.

(b) Suppose that $f : S \to T$ is onto. Prove that f is one-to-one.