Day 5 homework - Assigned 1/24 and due 1/31

Starred problems below are extra-credit for undergraduates and required for graduate students.

3. Let S be the set of positive rational numbers (that is, numbers of the form a/b where a and b are both positive integers). Define $x * y = x/y$. Which of the following properties does this operation have: closure, associativity, identity, inverses?

4. Let G be the set of 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ where a and c are nonzero real numbers, and b is any real number. Show that G is a group (where the operation is matrix multiplication).

5. * Let $S = \{1, 2, 3\}$. Find a set G consisting of two or more functions from S to itself so that G is a group (under function composition), and also so that no function in G is one-to-one.