

Day 6 homework - Assigned 1/27 and due 2/7

Starred problems below are extra-credit for undergraduates and required for graduate students.

1. Let $g = \begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix} \in GL(2, Z_5)$. Write $g^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c and d are in $\{0, 1, 2, 3, 4\}$.
2. Prove that if a and b are integers and $a \equiv b \pmod{n}$, then $\gcd(a, n) = \gcd(b, n)$.
3. Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} : a \in \mathbb{R} \text{ with } a \neq 0 \right\}$. Show that G is a group under matrix multiplication. Explain why each element of G has an inverse even though each matrix in G has a determinant of 0.