Day 7 homework - Assigned 1/29 and due on 2/7.

Starred problems below are extra-credit for undergraduates and required for graduate students.

4. Let G be a group with the property that for any x, y and z in the group, $xy = zx$ implies that $y = z$. Prove that G is abelian.

5. Can we have a group G (with identity e) containing a non-identity element a satisfying $a^3 = e$ so that the inverse b of a satisfies $b^2 = e$? Why or why not?

6. Justify your answer and show your work on this question.

(a) What's the order of 7 in $U(100)$?

(b) What's the order of $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ in $GL(2, Z_3)$?

(c) If G is a group containing elements a and b so that $|a| = 2$, $|b| = 3$ and $ab = ba$, what is $|ab|$?