Day 8 homework - Assigned 1/31 and due on 2/7

Starred problems below are extra-credit for undergraduates and required for graduate students.

7. There are three proper subgroups of $U(14)$. What are they? Describe how you found your answer, and why the sets you give are the three proper subgroups. You don't need to give a formal proof that the sets are subgroups, or that there aren't any more subgroups.

8.

(a) Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Define G to be the set of elements M in $GL(2, \mathbb{R})$ so that $M^T AM = A$. (Here if $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $M^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is the transpose of M.) Show that G is a subgroup of $GL(2, \mathbb{R})$.

(b) Let G be a group that contains an element c of order 3. Define H to be the set of elements x in G so that xcx^{-1} is equal to one of c or c^{-1}. Prove that H is a subgroup of G.

9. $*$ Let G be a group with elements $a \neq e$ and b so that b has order 7 and $bab^{-1} = a^2$. What's the order of a? (On this problem, you may use Corollary 2 from Chapter 4 which states that if G is a group and a an element of G with order n, then if $a^k = e$, then n divides k.)