Day 9 homework - Assigned 2/3 and due on 2/14.

Starred problems below are extra-credit for undergraduates and required for graduate students.

1. Suppose that G is a group and a is an element in G with finite order. Prove that $|a| = |\langle a \rangle|$. (That is, show that the order of a is equal to the order of the subgroup generated by a.) [If you need a hint, this is Corollary 1 from Chapter 4. The ideas involved are similar to the proof of the finite subgroup test though.]

2. Show that $U(14) = \langle 3 \rangle = \langle 5 \rangle$ (and so $U(14)$ is cyclic). Is $U(14) = \langle 11 \rangle$?

3. Let G be a group and H a subgroup of G. For $x \in G$, define xHx^{-1} to be the set of all elements of the form xhx^{-1} (where h ranges over all elements of H).

Define $N(H) = \{x \in G : xHx^{-1} = H\}$. Show that $N(H)$ is a subgroup of G.