Conservation of mechanical energy

If there are only conservative forces and the system is isolated (no energy added or removed) then, the total mechanical energy of a system remains constant.

\[ E = K + U \]

The final and initial energy of a system remain the same: \( E_i = E_f \)

Hence,

\[ E = K_i + U_i = K_f + U_f \]

Potential energy Plots

If we plot the potential energy and mechanical energy is conserved, then we can graphically obtain information about the motion of particles.

\( K = E - U(x) \), so the kinetic energy varies as a function of \( x \).

Conceptual Example

Conversion of elastic potential energy into kinetic energy
Example

A mass (m = 5 kg) is dropped from a height of 0.82 m onto a spring with spring constant k = 10 N/m.

(a) What is the speed of the mass right before it hits the spring.

(b) To what maximum height will the block recoil?

(c) If the block has an initial downwards speed of v = 10 m/s to what height will the block recoil after it bounces of the spring?

Assume there are no frictional losses.

Example II

The launching mechanism of a toy gun consists of a spring of unknown spring constant. When the spring is compressed 0.120 m, the gun is able to launch a 0.035 kg projectile to a max. height of 20.0 m.

(a) Neglecting all resistive forces determine the spring constant.

(b) What is the speed of the projectile as it moves through the equilibrium position of the spring?

(c) What is the speed at a height of 10 m.

External Forces and Energy

Work is transferred to or from a system by means of external forces.

If there is no friction, then

\[ W = \Delta K + \Delta U = \Delta E_{mech} \]

If there is friction, then there is an additional component

\[ W = \Delta E_{mech} + \int f_x d = \Delta E_{mech} + \Delta E_{thermal} \]
Conservation of energy

The total energy of a system can change only by transferring energy into or out of the system.

\[ W = \Delta E = \Delta E_{\text{mech}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \]

If there a system is isolated therefore the energy cannot change!

Hence (for an isolated system),

\[ 0 = \Delta E_{\text{mech}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \]

Example III

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210.

(a) How far does she travel on the horizontal stretch.

(b) Assume the slope also has a coefficient of friction equals 0.210. How far does she travel now?