Elastic collisions

Momentum is conserved.

\[ \vec{p}_i = \vec{p}_f \]

Kinetic Energy is conserved.

\[ KE_i = KE_f \]

\[ m_1v_{1i}^2 + m_2v_{2i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \]

So in an elastic collision, particles bounce off each other without loss of kinetic energy.

Usually such collisions are approximate!

Quiz

You are given two carts, A and B. They look identical, and you are told that they are made of the same material. You place A at rest on an air track and give B a constant velocity directed to the right so that it collides elastically with A. After the collision, both carts move to the right, the velocity of B being smaller than what it was before the collision. What do you conclude?

1. Cart A is hollow.
2. The two carts are identical.
3. Cart B is hollow.

Inelastic collisions

Momentum is conserved.

\[ \vec{p}_i = \vec{p}_f \]

Kinetic Energy is NOT conserved.

\[ m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \]

So in an inelastic collision, particles bounce off each other with a loss of kinetic energy!

The lost kinetic energy is converted into thermal or internal energy.

A completely inelastic collision is one where the particles stick together. (also called a perfectly inelastic collision)
Elastic versus Inelastic collisions

Momentum is conserved (unless there is an external force)

Kinetic Energy is conserved only in an elastic collision

Elastic:

\[ p_i = p_f \]
\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]
\[ m_1 \vec{v}_{1i}^2 + m_2 \vec{v}_{2i}^2 = m_1 \vec{v}_{1f}^2 + m_2 \vec{v}_{2f}^2 \]

Inelastic:

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]
\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]
\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

Perfectly inelastic collision of two particles

Before collision

\[ m_1 \quad \vec{v}_{1i} \]
\[ m_2 \quad \vec{v}_{2i} \]

After collision

\[ m_1 + m_2 \quad \vec{v}_f \]

Notice that \( \vec{p} \) and \( \vec{v} \) are vectors and, thus have a direction (+/-)

\[ K_i = K_f + E_{\text{loss}} \]
\[ \frac{1}{2} m_1 \vec{v}_{1i}^2 + \frac{1}{2} m_2 \vec{v}_{2i}^2 = \frac{1}{2} (m_1 + m_2) \vec{v}_f^2 + E_{\text{loss}} \]

There is a loss in energy \( E_{\text{loss}} \)

Perfectly inelastic collision of two particles II

Before collision

\[ m_1 \quad \vec{v}_{1i} \]
\[ m_2 \quad \vec{v}_{2i} \]

After collision

\[ m_1 + m_2 \quad \vec{v}_f \]

What if one particle was initially stationary?

\[ \vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2} \]

\[ \vec{v}_f = \frac{m_2 \vec{v}_{2i}}{m_1 + m_2} \]
Example I

Ballistic Pendulum:

In a ballistic pendulum a bullet (0.1 kg) is fired into a block (0.5 kg) that is suspended from a light string. The block (with the bullet stuck in it) is lifted up by 0.051 m.

(a) What is the speed of the combined bullet/pendulum right after the collision?
(b) Find the initial speed of the bullet?
(c) Find the loss in mechanical energy due to the collision

Example 2

Accident investigation. Two automobiles of equal mass approach an intersection. One vehicle is traveling towards the east with 29 mi/h (13.0 m/s) and the other is traveling north with unknown speed. The vehicles collide in the intersection and stick together, leaving skid marks at an angle of 55° north of east. The second driver claims he was driving below the speed limit of 35 mi/h (15.6 m/s).

Is he telling the truth?
What is the speed of the “combined vehicles” right after the collision?
How long are the skid marks (\( \mu_k = 0.5 \))

Quiz

A cart moving at speed \( v \) collides with an identical stationary cart on an air track, and the two stick together after the collision. What is their velocity after colliding?

1. \( v \)
2. 0.5 \( v \)
3. zero
4. \( -0.5 \) \( v \)
Quiz

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

1. Yes
2. No because such an occurrence would violate the law of conservation of momentum.