Quiz

A cart moving at speed $v$ collides with an identical stationary cart on an airtrack, and the two stick together after the collision. What is their velocity after colliding?

1. $v$
2. $0.5v$
3. zero
4. $-0.5v$

Example

**Accident investigation.** Two automobiles of equal mass approach an intersection. One vehicle is traveling towards the east with 29 mi/h (13.0 m/s) and the other is traveling north with unknown speed. The vehicles collide in the intersection and stick together, leaving skid marks at an angle of 55° north of east. The second driver claims he was driving below the speed limit of 35 mi/h (15.6 m/s).

Is he telling the truth?
What is the speed of the “combined vehicles” right after the collision?
How long are the skid marks ($\mu_k = 0.5$)?

Elastic collisions in one dimension

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$
Two-dimensional collisions

Conservation of momentum:

\[ m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \]

Split into components:

\[ m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx} \]
\[ m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy} \]

If the collision is elastic, we can also use conservation of kinetic energy.

Elastic collisions

Momentum is conserved.

\[ \vec{p}_i = \vec{p}_f \]

Kinetic Energy is conserved.

\[ KE_i = KE_f \]
\[ m_1v_{1i}^2 + m_2v_{2i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2 \]

So in an elastic collision, particles bounce off each other without loss of kinetic energy.

Usually such collisions are approximate!

Linear Momentum and Impulse

The impulse is defined as:

\[ \vec{J} = \int_{t_i}^{t_f} \vec{F} dt \]

where \( \vec{F} \) is the force of the collision.

\[ \vec{J} = \int_{t_i}^{t_f} \vec{F} dt = \Delta \vec{p} \]

The impulse is the change in momentum due to a collision.
If the object has a uniform density, then the integrals simplify:

\[
\bar{x}_{\text{COM}} = \frac{\int dV_x}{V} \quad \bar{y}_{\text{COM}} = \frac{\int dV_y}{V} \quad \bar{z}_{\text{COM}} = \frac{\int dV_z}{V}
\]

If the object has symmetry, then the CM lies on/at the symmetry element, and so at least one integral can be done automatically!

Sphere -> a point of symmetry: the center so the CM is at the center
Cone -> a line of symmetry: the center so the CM is along the line of symmetry

Center of mass: More General Case

Center of mass for many particles in multiple dimensions

\[
\vec{r}_{\text{CM}} = \frac{\sum m_i \vec{r}_i}{M}
\]

Note: the center of mass (CM) has components!

Review of energy and work

Work:
\[
W = \vec{F} \cdot \vec{d} = F \cdot \cos \theta = F_x \cdot d_x + F_y \cdot d_y + F_z \cdot d_z
\]

\[
W = \int F(x) \, dx
\]

Energy:

Kinetic energy: \( K = \frac{1}{2} m \cdot v^2 \)
Gravitational potential energy: \( U_g = m \cdot g \cdot y \)
Elastic potential energy: \( U_e = \frac{1}{2} k x^2 \)
Conservation of energy
The total energy of a system can change only by transferring energy into or out of the system.

\[ W = \Delta E = \Delta E_{\text{mech}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \]

If there is a system is isolated therefore the energy cannot change!
Hence (for an isolated system),

\[ 0 = \Delta E_{\text{mech}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \]

Conservation of mechanical energy
If there are only conservative forces and the system is isolated (no energy added or removed) then,
the total mechanical energy of a system remains constant.

\[ E = K + U \]

The final and initial energy of a system remain the same: \( E_i = E_f \)
Hence,

\[ E = K_i + U_i = K_f + U_f \]

Potential energy Plots
If we plot the potential energy and mechanical energy is conserved, then we can graphically obtain information about the motion of particles.

\[ E = U(x) \]

K=E-U(x), so the kinetic energy varies as a function of x.
Frictional Force

Two types:

- Static friction, $f_s$
  \[ f_s \leq \mu_s n \]

- Kinetic friction, $f_k$
  \[ f_k = \mu_k n \]

Force always opposes motion! Constants depend on the interface.