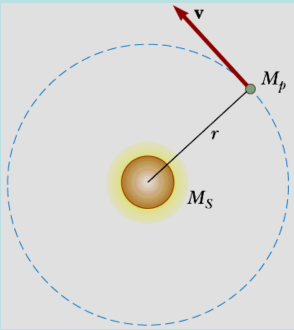
Kepler's laws about planetary motion



Most planets, except Mercury and Pluto, are on almost a circular orbit

Earth:

Ratio of minor to major axis b/a = 0.99986.

For planets around sun:

$$\frac{T^2}{a^3} = 2.97 \cdot 10^{-19} \frac{s^2}{m^3}$$

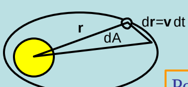
Kepler's Second law

Kepler's Second law:
The radius vector drawn from the sun to a planet sweeps out equal areas in equal times.

This is a consequence of angular momentum and the nature of gravity.

The gravitational force is parallel to **r**, and so there is no torque from gravity on an orbiting planet. Therefore angular momentum is conserved!

$\tilde{L} = \tilde{r} \times \tilde{p} = M(\tilde{r} \times \tilde{v})$ = constant



$$dA = \frac{|\tilde{r} \times d\tilde{r}|}{2} = \frac{|\tilde{r} \times \tilde{v} dt|}{2} = \frac{L dt}{2M} \Rightarrow \frac{dA}{dt} = \frac{L}{2M} = \text{constan}$$

Ponder the following: does this depend on the inverse-square law?

Kepler's Third law: Circular Orbits

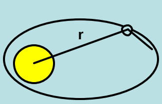
Kepler's Third law:
The square of the orbital period of a planet is proportional to the cube of the semimajor axis.

Let us consider this for circular orbits.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2pr}{T}$$

$$\frac{GM}{r^2} = \frac{\left(2pr/T\right)^2}{r}$$



$$T^2 \propto r^3$$

Fluids

A fluid is anything which will flow, and conform to the shape of a container

A fluid cannot withstand a shear force.

Liquids and gases are all fluids!

Density: Reminder

Density is the ratio between mass and volume:

$$r = \frac{m}{V}$$ For uniform substances

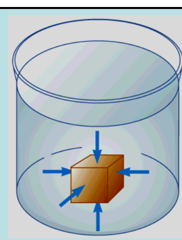
$$r = \frac{\Delta m}{\Delta V} \Rightarrow r(\tilde{r}) = \frac{\partial m}{\partial V}$$ More generally

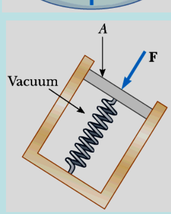
Pressure

$$P \equiv \frac{F}{A}$$

F… force

A… area

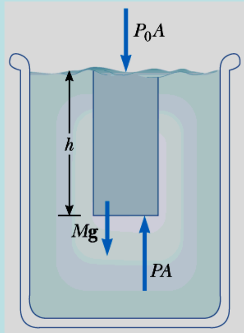




Unit of pressure:

1 Pascal; 1Pa = 1 N/m^2

Variation of pressure with depth



$$P = P_0 + r \cdot g \cdot h$$

The pressure P at a depth h below the surface of a liquid open to the atmosphere is *greater* then the atmospheric pressure by an amount ρ·g·h

ρ… density of liquid

i.e. added pressure corresponds to weight of fluid column of height h.

Quiz I

When a hole is made in the side of a stationary container holding water, the water:

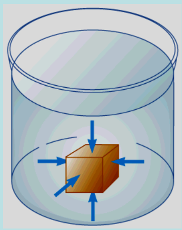
1) flows out in a straight line.
2) flows out in a parabolic trajectory.
3) stays inside the container.
4) sprays in all directions.

Quiz II

. When a hole is made in the side of a container holding water, water flows out and follows a parabolic trajectory. If the container is dropped in free fall, the water flow

1. diminishes.
2. stops altogether.
3. goes out in a straight line.
4. curves upward.

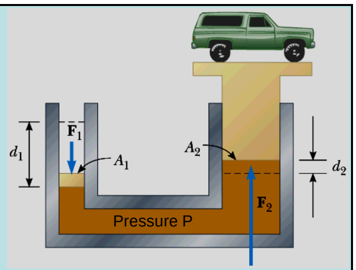
Pascal's law: A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.



Example

Hydraulic press

Application of Pascal's law



- Force F_1 is applied to area A_1
- Pressure P in columns: $P = F_1/A_1 = F_2/A_2$
- Force F_2 on area A_2 is greater than F_1 by a factor A_2/A_1!!