Review

• Instantaneous acceleration – the derivative of velocity with respect to time; also the double time derivative of displacement
• Constant acceleration – special (but important) case for which (with the help of calculus) we can derive special equations
• Free Fall – fall under gravity; important example of constant acceleration under $F=-g$

\[
\begin{align*}
\dot{v}_t &= v_0 + at \\
x &= x_0 + v_0t + \frac{1}{2}at^2 \\
p^2 &= v_0^2 + 2a(x-x_0)
\end{align*}
\]

Coordinate systems

Different ways of representing space, and physics. Some problems are easier in some coordinate systems, but the physics is invariant.

Cartesian Coordinates:

\[
n = (x, y) \quad (-3, 4) \quad (0, 3)
\]

Polar Coordinates

Another popular coordinate system, along with cylindrical and spherical

\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta \\
\tan \theta &= \frac{y}{x} \\
r &= \sqrt{x^2 + y^2}
\end{align*}
\]
**Vectors and Scalars**

- **Vectors**: Magnitude and direction
- **Scalars**: Magnitude

Displacement is a vector. Velocity is a vector. Acceleration is a vector.

- **Arrows**
  - Tip points away from the starting point.
  - Length of the arrow represents the magnitude
  - Various different symbols.

These four vectors are equal because they have the same magnitude and same direction

**Representations of Vectors**

- **Arrows**
- Tip points away from the starting point.
- Length of the arrow represents the magnitude
- Various different symbols.

Vector: $\vec{A}, \vec{A}, \vec{A}, \vec{A}, \vec{A}$

Magnitude: $|\vec{A}|$

**Vector Addition: Geometric**

- Draw vector $\vec{A}$.
- Draw vector $\vec{B}$ starting at the tip of vector $\vec{A}$.
- The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is drawn from the tail of $\vec{A}$ to the tip of $\vec{B}$.
Resultant vector $R = A + B + C + D$ is drawn from the tail of the first vector to the tip of the last vector.

The negative of a vector has the same magnitude but opposite sign.

A car travels 20.0 km due north and then 35.0 km in a direction 60° west of north. Find the magnitude and direction of the car’s resultant displacement.

We cannot just add 20 and 35 to get resultant vector!!
The x- and y-components of a vector:
\[ A_x = A \cos \theta \]
\[ A_y = A \sin \theta \]

The magnitude of a vector:
\[ A = \sqrt{A_x^2 + A_y^2} \]

The angle \( \theta \) between vector and x-axis:
\[ \theta = \tan^{-1}\left( \frac{A_y}{A_x} \right) \]

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**Vector Components: Geometric**

**Vector Components: Algebraic**

- A unit vector is a dimensionless vector having a magnitude 1.
- Unit vectors are used to indicate a direction.
- \( \hat{i}, \hat{j}, \hat{k} \) represent unit vectors along the x-, y- and z-direction.
- \( \hat{x}, \hat{y}, \hat{z} \) is another common notation.
- \( \hat{i}, \hat{j}, \hat{k} \) form a right-handed coordinate system.

**Vector Components: Geometric**

**Vector Components: Algebraic**

\[ \mathbf{A} = A_x \hat{i} + A_y \hat{j} \]

**Vector Addition: Algebraic I**

We want to calculate: \( \mathbf{R} = \mathbf{A} + \mathbf{B} \)

From diagram:
\[ \mathbf{R} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) \]
\[ \mathbf{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} \]

The components of \( \mathbf{R} \):
\[ R_x = A_x + B_x \]
\[ R_y = A_y + B_y \]
The magnitude of $\mathbf{R}$:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

The angle $\theta$ between vector $\mathbf{R}$ and x-axis:

$$\tan \theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$$

Vector Multiplication
There are two ways (in 2 or 3D) to multiply vectors.

Scalar product -> two vectors make a scalar

$$\overrightarrow{A} \cdot \overrightarrow{B} = N$$
Also called the dot product or the inner product

Vector product -> two vectors make a scalar

$$\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{C}$$
Also called the cross product or the outer product

Scalar Product
Scalar product -> two vectors make a scalar

$$\overrightarrow{A} \cdot \overrightarrow{B} = ab \cos \theta$$
Geometric

$$\overrightarrow{A} \cdot \overrightarrow{B} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
Algebraic
Vector Product

Vector product -> two vectors make a vector

Geometric

\[ \vec{A} \times \vec{B} = \vec{C} \]

C has magnitude \( \text{absin}\theta \), Direction perpendicular to the plane containing \( \vec{A} \) and \( \vec{B} \).

Algebraic

\[ \vec{A} \times \vec{B} = (a_yb_z - a_zb_y)i + (a_zb_x - a_xb_z)j + (a_xb_y - a_yb_x)k \]