Chapter 10 Hint for Exercise 2

The total impulse of the abutment on the car is $-F_{\text{avg}}\,\Delta t$, where F_{avg} is the average force and Δt is the duration of the collision. The forward direction was taken to be the positive direction. According to the impulse-momentum theorem the total impulse gives the change in the linear momentum of the car. Since the collision stops the car, its final linear momentum is zero. Its initial linear momentum is mv_i, where m is its mass and v_i is its initial speed. Thus

$$-F_{\text{avg}}\,\Delta t = -mv_i\,.$$

Solve for F_{avg}.

Suppose ΔN bullets strike Superman in time Δt. If F_{avg} is the magnitude of the average force of Superman on the bullets then the total impulse on these bullets is $-F_{\text{avg}}\,\Delta t$. The initial direction of motion of the bullets is taken to be the positive direction. According to the impulse-momentum theorem the total impulse gives the change in the total linear momentum of the ΔN bullets. Initially the total linear momentum is $\Delta N\,mv$, where v is the speed of each bullet and m is the mass of a bullet. The final total linear momentum is $-\Delta N\,mv$ and the change in the total linear momentum is $-2\Delta N\,mv$. Thus

$$-F_{\text{avg}}\,\Delta t = -2\Delta N\,mv\,.$$

Solve for F_{avg}. Convert $m = 3\,\text{g}$ to kilograms and $\Delta N/\Delta t = 100\,\text{bullets/min}$ to bullets per second.

According to Newton's third law the average force of the bullets on Superman man has the same magnitude as the average force of Superman on the bullets.

Calculate the impulse by finding the area under the curve. Your result will be in terms of F_{max}. Equate this to the magnitude of the change in linear momentum, which is $2mv$, where v is the speed of the ball. Solve for F_{max}.

The area of a triangle is half the base times the altitude so the impulse from $t = 0$ to $t = 2\,\text{ms}$ is $(2 \times 10^{-3}\,\text{s})F_{\text{max}}/2$. The area of a rectangle is the product of its base and altitude so the impulse from $2\,\text{ms}$ to $4\,\text{ms}$ is $(2 \times 10^{-3}\,\text{s})F_{\text{max}}$. Finally, the impulse from $4\,\text{ms}$ to $6\,\text{ms}$ is $(2 \times 10^{-3}\,\text{s})F_{\text{max}}/2$. The total impulse is the sum of these, or $(4 \times 10^{-3}\,\text{s})F_{\text{max}}$. In each case the impulse is in N·s for F_{max} in N.

Chapter 10 Hint for Problem 16

(a) The initial and final speeds are related by $\frac{1}{2}mv_f^2 = \frac{1}{2}\frac{1}{2}mv_i^2$ so $v_f = v_i/\sqrt{2}$.

(b) The impulse of the ball on the wall is equal in magnitude and opposite in direction to the impulse of the wall on the ball. If we assume the ball hits the wall perpendicularly and bounces straight back, that impulse is given by

$$J = p_f - p_i = m(v_f - v_i)\,.$$

Don't forget that these velocities have opposite signs. If the positive direction is toward the wall, v_i is positive and v_f is negative.

(c) Use $J = F_{\text{avg}}\,\Delta t$.

CHAPTER 10 HINT FOR EXERCISE 20

(a) Assume that the only horizontal forces acting are the force of the bullet on the block and the force of the block on the bullet. These are internal to the bullet-block system, so the horizontal component of the total linear momentum of that system is conserved.

Initially the linear momentum of the block is zero and the linear momentum of the bullet is $m_{\text{bull}} v_{\text{bull}\ i}$, where m_{bull} is the mass of the bullet and $v_{\text{bull}\ i}$ is its initial speed. The forward direction is taken to be positive.

After the bullet passes through the block its linear momentum is $m_{\text{bull}} v_{\text{bull}\ f}$, where $v_{\text{bull}\ f}$ is its speed. The linear momentum of the block is $m_{\text{block}} v_{\text{block}\ f}$, where m_{block} is the mass of the block and $v_{\text{block}\ f}$ is its speed.

Conservation of linear momentum yields

$$m_{\text{bull}} v_{\text{bull}\ i} = m_{\text{bull}} v_{\text{bull}\ f} + m_{\text{block}} v_{\text{block}\ f}\,.$$

Solve for $v_{\text{block}\ f}$

(b) Since the horizontal component of the total linear momentum of the system is conserved, the horizontal component of the velocity of its center of mass is constant; it does not change during the collision. Use

$$v_{\text{com}} = \frac{m_{\text{bull}} v_{\text{bull}} + m_{\text{block}} v_{\text{block}}}{m_{\text{bull}} + m_{\text{block}}}\,,$$

where the velocities are evaluated either for a time before the collision or for a time after the collision.

Consider the system consisting of the bullet and the two blocks. Those objects exert forces on each other but the forces are internal to the system. There are no external forces with horizontal components. This means that the horizontal component of the total linear momentum of the system is conserved.

Let m_b be the mass of the bullet, m_1 be the mass of the first block struck, and m_2 be the mass of the second block struck. Let v_0 be the initial speed of the bullet, v_b be its speed after it pass through the first block, and v_2 be its speed (and the speed of the second block) after the bullet becomes embedded in the second block. Let v_1 be the final speed of the first block. Conservation of linear momentum, applied to the first collision, yields

$$m_b v_0 = m_b v_b + m_1 v_1$$

and applied to the second collision yields

$$m_b v_b = (m_b + m_2) v_2 \,.$$

The second equation can be solved for v_b and the first can be solved for v_0.

Chapter 10 Hint for Problem 33

Take the system to be comprised of the two blocks and the spring. When the compression of the spring is maximum, the two blocks have the same velocity and conservation of linear momentum for the system yields

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \,,$$

where m_1 and m_2 are the masses of the blocks, v_{1i} and v_{2i} are their initial velocities, and v_f is their final velocity. Solve for v_f. Whatever kinetic energy was lost by the blocks is stored as potential energy in the spring. If x is the compression of the spring, then conservation of mechanical energy yields

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2}(m_1 + m_2) v_f^2 + \frac{1}{2} k x^2 \,.$$

Solve for x.

[ans: 25 cm]

Chapter 10 Hint for Problem 34

Assume that the blocks do not move a significant distance during the collision, which lasts until the heavier block becomes fastened to the lighter. Each block exerts a force on the other but these forces are internal to the two-block system and there are no external forces with horizontal components, so the horizontal component of the total linear momentum of the two-block system is conserved.

Let m_h be the mass of the heavier block and v_0 be its initial speed. Let m_ℓ be the mass of the lighter block and V be the speed of the two blocks immediately after the collision, when they are fastened together. Conservation of linear momentum yields

$$m_h v_0 = (m_h + m_\ell)V\,.$$

Note that mechanical energy is not conserved during the collision.

After the collision the blocks move to the right and compress the spring. Because the spring exerts a force on them, their total linear momentum is not conserved as they move. However, the spring force is a conservative force and the mechanical energy of the two blocks and the spring, taken together, is conserved.

Take the initial elastic potential energy, when the spring has its relaxed length, to be zero. Then when the blocks come to rest and the spring is compressed a distance x, the potential energy is $U = \frac{1}{2}kx^2$, where k is the spring constant. The initial kinetic energy, just after the blocks stick together, is $\frac{1}{2}(m_h + m_\ell)V^2$ and the final kinetic energy is zero. Conservation of mechanical energy yields

$$\frac{1}{2}(m_h + m_\ell)V^2 = \frac{1}{2}kx^2\,.$$

Use the conservation of linear momentum equation to substitute for V, then solve for x.

Chapter 10 Hint for Problem 40

You need to know the speed of the ball just before it strikes the block. Use conservation of mechanical energy, applied to the system consisting of the ball and Earth. The time is when the ball starts to drop and the final time is just before the ball strikes the block. If the potential energy is zero just before the ball strikes the block, the initial potential energy is $m_{\text{ball}}gL$, where L is the length of the cord. The initial kinetic energy is zero and the final kinetic energy is $\frac{1}{2}m_{\text{ball}}v^2$. Solve

$$m_{\text{ball}}gL = \frac{1}{2}m_{\text{ball}}v^2$$

for v. You should get $3.70\,\text{m/s}$. This is the initial velocity for the collision. Since the collision is elastic and the block is initially at rest, the final velocity of the ball is given by

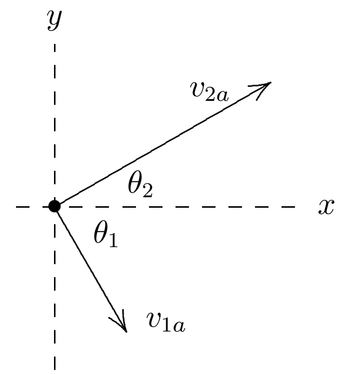
$$v_{\text{ball}} = \frac{(m_{\text{ball}} - m_{\text{block}})}{m_{\text{ball}} + m_{\text{block}}}v$$

and the final velocity of the block is given by

$$v_{\text{block}} = \frac{2m_{\text{ball}}}{m_{\text{ball}} + m_{\text{block}}}v\,,$$

where m_{block} is the mass of the block.

The situation after the collision is diagramed on the right. The projectile proton enters the collision along the x axis and leaves at an angle $\theta_1 = 60°$ below that axis. Its initial speed is v_{1b} ($= 500\,\mathrm{m/s}$) and its final speed is v_{1a}. The target proton leaves the collision at an angle $\theta_2 = 30°$ above the x axis. Its initial speed is zero and its final speed is v_{2a}. Note that the angle between the two paths is 90°, as demanded by the problem statement. Since momentum is conserved



$$\vec{p}_{1b} = \vec{p}_{1a} + \vec{p}_{2a}\,.$$

The x component of this equation is

$$mv_{1b} = mv_{1a}\cos\theta_1 + mv_{2a}\cos\theta_2$$

and the y component is

$$0 = -mv_{1a}\sin\theta_1 + mv_{2a}\sin\theta_2\,.$$

Solve these equations simultaneously for (a) v_{2a} and (b) v_{1a}. The second equation yields

$$v_{1a} = \frac{\sin\theta_2}{\sin\theta_1}\,v_{2a}\,.$$

This expression is used to substitute for v_{1a} in the first equation. The result is solved for v_{2a}. You should get

$$v_{2a} = \frac{\sin\theta_1}{\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1}\,v_{1b}\,.$$

Evaluate this result, then evaluate the expression for v_{1a}.

(a) The conservation of linear momentum equations are

$$m_B v_{Bi} = m_A v_{AF} \cos\theta_A$$

and

$$0 = m_A v_{Af} \sin\theta_A - m_B v/2\,,$$

where the scattering angles θ_A and θ_B are on opposite sides of the original direction of motion of B. Solve for θ_A.

One way to do this is to use one of the equations to eliminate v_{AF} from the other. You should get $\tan\theta_A = 1/2$.

(b) Solve the linear momentum conservation equations for v_{Af}. You should get

$$v_{Af} = \frac{m_B v}{2m_A \sin\theta_A}\,.$$

What must you be given to obtain a value for v_{Af}?

(a) Label the incident ball 1 and the target ball 2. Then, the conservation of linear momentum equations are

$$v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2$$

and

$$0 = v_{1f}\sin\theta_1 - v_{2f}\sin\theta_2\,,$$

where θ_2 is on the opposite side of the original line of motion from θ_1. Solve these simultaneously for v_{2f} and θ_2.

First eliminate θ_2. Solve the first equation for $\cos\theta_2$ and the second for $\sin\theta_2$. Square the results and add them. Use $\cos^2\theta_2 + \sin^2\theta_2 = 1$ and obtain

$$v_{2f} = \sqrt{v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta_1 + v_{1f}^2}\,.$$

After evaluating this expression use $\sin\theta_2 = (v_{1f}/v_{2f})\sin\theta_1$ to find θ_2.

(b) Compare the kinetic energy before the collision with the kinetic energy after. Since the masses are the same all you need to do is compare v_{1i}^2 with $v_{1f}^2 + v_{2f}^2$.