Chapter 12 Hint for Problem 2

(a) through (f) As viewed by a passenger, the tire is simply rotating. Every point on the rim has speed $v = v_c$, where v_c is the speed of the car. The point on top is moving forward, the point on the bottom is moving backward. Their accelerations have magnitude v^2/R, toward the center. The velocity and acceleration of the center of the wheel are zero.

(g) and (l) Simply add vectorially $\vec{v}_c$, in the forward direction, to each of the velocities found above. Since the observer is not accelerating with respect to the car, the accelerations are the same.

(a) Here an acceleration is given, not a speed, so we use a dynamical method, not an energy method. Write Newton's second law equations for the motion of the center of mass and for rotation about the center of mass. Apply the condition $a_{\rm com} = \alpha R$, where $a_{\rm com}$ is the acceleration of the center of mass, α is the angular acceleration about the center of mass, and R is the radius. Solve for the incline angle.

Draw a force diagram for the sphere. Take the x axis to be parallel to the incline and down the incline. Take the y axis to be perpendicular to the incline. Draw the sphere and show the forces: the force of friction f, up the incline and acting at the point of contact of the sphere with the plane; the normal force N, in the y direction and acting at the point of contact, and the force of gravity mg, down and acting at the center of the sphere. Let θ be the angle of incline and write the x component of the second law:

$$mg\sin\theta - f = ma_{\rm com}\,,$$

where m is the mass of the sphere and $a_{\rm com}$ is its acceleration.

Choose the direction for positive rotation, consistent with rolling down the plane, and write the second law for rotation:

$$fR = I\alpha\,,$$

where I is the rotational inertia of the sphere. The normal force and the force of gravity do not exert torques about the center of the sphere; only the force of friction does. Finally write the condition for no sliding:

$$a_{\rm com} = \alpha R\,,$$

where R is the radius of the sphere. Systematically eliminate α and f, then solve for $\sin\theta$. You should get

$$\sin\theta = (1 + I/mR^2)\frac{a_{\rm com}}{g}\,.$$

Substitute $I = \frac{2}{5}mR^2$ (see Table 11–2) and calculate θ.

(b) Use Newton's second law and solve for $a_{\rm com}$ or put $I = 0$ in the result for the sphere. You should get

$$a_{\rm com} = g\sin\theta\,.$$

Your result should be greater than $0.1g$. To understand why, think about the forces on the rolling sphere that have components along the incline.

[ans: (a) 8.1°; (b) more]

CHAPTER 12 HINT FOR PROBLEM 8

(a) Draw a force diagram for the wheel. Four forces act on it : the applied force F, at the center and directed to the right, the frictional force f, at the point of contact with the surface and directed to the left, the force of gravity mg, at the center and directed downward, and the normal force of the surface, at the point of contact with the surface and directed upward.

Take the positive direction to be to the right. Then the horizontal component of Newton's second law is

$$F - f = ma_{\text{com}}\,,$$

where m is the mass of the wheel and a_{com} is the acceleration of its center of mass. Solve for f. If you get a positive result then the frictional force is indeed to the left. If you get a negative result it is to the right.

(b) The angular acceleration of the wheel about its center is

$$\alpha = -\frac{a_{\text{com}}}{R}\,,$$

where counterclockwise was chosen to the positive. Here R is the radius of the wheel. According to Newton's second law for rotation

$$\tau_{\text{net}} = I\alpha\,,$$

where τ_{net} is the net torque on the wheel and I is the rotational inertia of the wheel. The only torque about the center of the wheel is the torque of friction and this is $-fR$, so

$$-fR = -I\frac{a_{\text{com}}}{R}\,.$$

Solve for I.

(a) First find the speed that the marble must have at the top in order to stay on the track. At the top the marble experiences the smallest possible inward force when the normal force of the track is zero and the only force acting is the force of gravity mg. The inward force is greater than this if the track exerts a normal force but it cannot be less than this. Thus, the minimum speed v of the center of mass at the top is determined by

$$mg = \frac{mv^2}{R-r}\,.$$

Note that the radius of the circle traversed by the center of mass, $R-r$, is used.

Now, use conservation of mechanical energy to find the initial height h of the center of mass that will result in speed v at the top. If the zero of gravitational potential energy is at the bottom of the loop, then the initial potential energy is mgh. The initial kinetic energy is 0 since the marble starts from rest. The potential energy at the top of the loop is $mg(2R-r)$ since the center of mass is then $2R-r$ above the bottom. The kinetic energy is given by $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where I is the rotational inertia of the marble. When the condition $R \gg r$ is used and when $I = \frac{2}{5}mr^2$ (see Table 11–2) and $\omega = v/r$ (no sliding) are substituted, the conservation of mechanical energy equation becomes

$$mgh = (7/10)mv^2 + 2mgR\,.$$

Use $mv^2 = mgR$ to eliminate v, then solve for h.

(b) When the marble is at Q the horizontal component of Newton's second law is $N = mv^2/R$, where v is the speed of the marble and N is the normal force of the track on the marble. $R \gg r$ was assumed. Use conservation of mechanical energy, with $h = 6R$, to find v.

Draw a force diagram for the ball. Three forces act on it: the force of gravity mg, down and at the center of the ball, the normal force of the lane N, up and at the point of contact with the lane, and the frictional force f, backward and at the point of contact with the lane.

Take the forward direction to be positive. Then the horizontal component of Newton's second law for the center of mass yields

$$-f = ma_{\text{com}}$$

and the vertical component yields

$$N - mg = 0\,.$$

The only torque about the center of the ball is the frictional torque. Its magnitude is fR, where R is the radius of the ball, and it is clockwise (the negative direction). Newton's second law for rotation about the center of the ball is

$$-fR = I\alpha\,,$$

where I is the rotational inertia of the ball about its center and α is the angular acceleration of the ball.

(a) Notice that the acceleration of the center of mass of the ball is directed opposite the velocity. The ball slows as it slides. At the same time the frictional torque causes the angular speed of the ball to increase. The ball stops sliding and starts to roll smoothly when $\omega R + v_{\text{com}} = 0$.

(b) and (c) While the ball is sliding the frictional force is kinetic in nature, so $f = \mu_k N$, where μ_k is the coefficient of kinetic friction. Substitute this expression into the horizontal component of Newton's second law for the center of mass and into Newton's second law for rotation, then solve for a_{com} and α. Also use $I = (2/5)mR^2$, which you can find in Table 11–2 of the text.

(d) Use

$$v_{\text{com}} = v_0 + a_{\text{com}}t$$

to find the velocity of the center of mass as a function of time and

$$\omega = \alpha t$$

to find the angular velocity as a function of time. Solve

$$\omega R + v_{\text{com}} = 0$$

for the time t when the rolling becomes smooth.

(e) Use

$$x_{\text{com}} = v_0 t + \frac{1}{2} a_{\text{com}} t^2 \,,$$

where t has the value found in part (d).

(f) Use

$$v_{\text{com}} = v_0 + a_{\text{com}} t \,,$$

where t has the value found in part (d).

(a) Evaluate

$$\vec{\tau} = \vec{r} \times \vec{F} \,.$$

Use

$$(y\,\hat{\jmath} + z\,\hat{k}) \times (F_x\,\hat{\imath} + F_z\,\hat{k}) = yF_z\,\hat{\imath} - zF_x\,\hat{\jmath} - yF_x\,\hat{k}\,.$$

(b) The position vector of the pebble relative to the point is given by

$$\vec{r}\,' = \vec{r} - \vec{R}\,,$$

where $\vec{R}$ is the position vector of the point relative to the origin. That is,

$$\vec{r}\,' = \Big[(0.50\,\mathrm{m})\,\hat{\jmath} - (2.0\,\mathrm{m})\,\hat{k}\Big] - \Big[(2.0\,\mathrm{m})\,\hat{\imath} - (3.0\,\mathrm{m})\,\hat{k}\Big] = (-2.0\,\mathrm{m})\,\hat{\imath} + (0.50\,\mathrm{m})\,\hat{\jmath} + (1.0\,\mathrm{m})\,\hat{k}\,.$$

Evaluate

$$\vec{\tau}\,' = \vec{r}\,' \times \vec{F}\,.$$

Use

$$\vec{r} \times \vec{F} = (x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}) \times (F_x\,\hat{\imath} + F_z\,\hat{k}) = yF_z\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} - yF_x\,\hat{k}\,.$$

Usually the torque around different points has different values. In this case the values are the same because the displacement of the second point from the origin is parallel to the force. That is, $\vec{R} \times \vec{F} = 0$.

CHAPTER 12 HINT FOR EXERCISE 24

(a) Use

$$\vec{\ell} = m\vec{r} \times \vec{v}$$

to calculate the angular momentum $\vec{\ell}$. Here $\vec{r}$ is the position vector of the particle (from the origin to the particle) and $\vec{v}$ is the velocity of the particle. The magnitude of the angular momentum is

$$\ell = mrv \sin\phi\,,$$

where ϕ is the angle between $\vec{r}$ and $\vec{v}$ when they are drawn with their tails at the same point. This is $\phi = 180° - 30° = 150°$. Use the right-hand rule to find the direction of $\vec{\ell}$.

(b) Use

$$\vec{\tau} = \vec{r} \times \vec{F}$$

o find the torque $\vec{\tau}$. The magnitude is

$$\tau = rF \sin\phi\,,$$

where ϕ is the angle between $\vec{r}$ and $\vec{F}$ when they are drawn with their tails at the same point. Use the right-hand rule to find the direction of $\vec{\tau}$.

Chapter 12 Hint for Problem 26

To find the angular momentum $\vec{\ell}$, use

$$\vec{\ell} = m\vec{r} \times \vec{v}\,,$$

where m is the mass of the particle, $\vec{v}$ is its velocity, and $\vec{r}$ is its position vector relative to the point about which the angular momentum is to be calculated. If x and y are the coordinates of the particle and x_0 and y_0 are the coordinates of the point, then

$$\vec{r} = (x - x_0)\,\hat{\imath} + (y - y_0)\,\hat{\jmath}\,.$$

In terms of components

$$\begin{aligned}\vec{\ell} &= m\,[(x - x_0)\,\hat{\imath} + (y - y_0)\,\hat{\jmath}] \times [v_x\,\hat{\imath} + v_y\,\hat{\jmath}] \\ &= m\,[(x - x_0)v_y - (y - y_0)v_x]\;\hat{k}\end{aligned}$$

This problem shows that the angular momentum of a particle may be different for different reference points.

(a) Evaluate

$$\vec{\ell} = m\vec{r} \times \vec{v}\,,$$

where $\vec{r}$ is the position vector of the particle and $\vec{v}$ is its velocity. Use

$$\vec{r} \times \vec{v} = (x\,\hat{\imath} + y\,\hat{\jmath}) \times (v_x\,\hat{\imath} + v_y\,\hat{\jmath}) = (xv_y - yv_x)\,\hat{k}\,.$$

(b) Use

$$\vec{\tau} = \frac{\mathrm{d}\vec{\ell}}{\mathrm{d}t}$$

to find an expression for the torque.

(c) The position vector of the particle relative to the new point is

$$\vec{r}\,' = [(4.0\,\mathrm{m})\,\hat{\imath} - (2.0\,\mathrm{m})\,\hat{\jmath})] - [(-2.0\,\mathrm{m})\,\hat{\imath} - (3.0\,\mathrm{m})\,\hat{\jmath}] = (6.0\,\mathrm{m})\,\hat{\imath} + (1.0\,\mathrm{m})\,\hat{\jmath}\,.$$

Evaluate

$$\vec{\ell}\,' = m\vec{r}\,' \times \vec{v}$$

and its derivative.

Notice that the angular momentum and torque have different values in (c) than in (a) and (b). Nevertheless, $\vec{\tau} = \mathrm{d}\vec{\ell}/\mathrm{d}t$ no matter what origin is used to calculate these quantities.

Chapter 12 Hint for Problem 36

The magnitude of the torque is $\tau(t) = F(t)R$ and the integral $\int F(t)\,\mathrm{d}t$ is $F_{\text{avg}}\Delta t$, so

$$\int \tau\,\mathrm{d}t = F_{\text{avg}} R\,\Delta t\,.$$

According to Newton's second law for rotation

$$\tau = I\alpha\,,$$

where I is the rotational inertia of the body and α is its angular acceleration. Since

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}\,,$$

where ω is the angular velocity,

$$\tau = I\,\frac{\mathrm{d}\omega}{\mathrm{d}t}\,.$$

You can now evaluate the integral $\int \tau\,\mathrm{d}t$ over some time interval and show it is $I(\omega_f - \omega_i)$, where ω_i is the angular velocity at the beginning of the interval and ω_f is the angular velocity at the end.

(a) The structure consists of four rods and one hoop. The total rotational inertia is the sum of the rotational inertias of the five objects.

The hoop is rotating about an axis that is tangent to it. Table 11–2 of the text tells us that the rotational inertia about a diameter is $I_{\text{com}} = \frac{1}{2}mR^2$ and the parallel axis theorem tells us that the rotational inertia for rotation about a tangent axis is

$$I_{\text{hoop}} = I_{\text{com}} + mR^2 = (3/2)mR^2 \,.$$

One bar is along the rotation axis. Its rotational inertia is $I_{\text{bar 1}} = 0$.

Two bars are rotating about their end points. According to Table 11–2 their rotational inertias are $I_{\text{bar 2}} = I_{\text{bar 3}} = (1/3)mR^2$.

The fourth bar is parallel to the rotation axis and a distance R from it. That is, all points on the bar are a distance R from the rotation axis, so its rotational inertia is $I_{\text{bar 4}} = mR^2$.

(b) The magnitude of the angular momentum of the structure is $L = I_{\text{total}}\omega$, where ω is the angular speed. The angular speed must be in radians per second, so $\omega = 2\pi/T$, where T is the period of the motion in seconds.

Use the right-hand rule to determine the direction of the angular momentum. Curl the fingers of your right hand around the rotation axis in the direction of rotation. Your thumb will then point in the direction of the angular momentum.

(a) Take the system to consist of the two disks. During the coupling process each exerts a torque on the other and their individual angular momenta change until they come to the same rotational speed. If friction in the bearings can be neglected, no external torques act on the system so the total angular momentum of the system is conserved during the coupling. Let I_1 be the rotational inertia and ω_1 be the original angular velocity of disk 1; let I_2 be the rotational inertia and ω_2 be the original angular velocity of disk 2. Then

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f \, ,$$

where ω_f is the rotational velocity of the coupled disks. Solve for ω_f. ω_1 and ω_2 have the same sign.

(b) The conservation of angular momentum equation is the same but now ω_1 and ω_2 have opposite signs.

(a) The net external force in the system consisting of the two skaters and the pole is zero. The forces of gravity on the skaters and the normal forces of the ice on them sum to zero. The forces of the skaters on the pole and the forces of the pole on the skaters are internal forces. Thus the center of mass of the system has a constant velocity. Since the skaters have equal masses and initially have velocities with equal magnitudes but in opposite directions the velocity of the center of mass is zero.

The net external torque is also zero, so the angular momentum of the system is conserved. The total angular momentum is not zero.

After both skaters are connected to the pole, they move on a circular path centered at their center of mass, with constant speeds. They are always diametrically opposite each other, with their velocities in opposite directions. This is the only motion for which the total linear momentum is zero but the total angular momentum is not. The radius of their circular path is half the original distance between them.

Before the skaters are connected to the pole, each has an angular momentum about the center of mass with magnitude $mvL/2$, where L is the original distance between them. Both angular momenta are out of the page in Fig. 12–41. After both skaters are connected to the pole and are traveling around a circle with radius $L/2$, each has an angular momentum of magnitude $m(L/2)^2\omega$, where ω is their angular speed. The angular momentum is again out of the page in the figure. Equate $2mvL/2$ and $2m(L/2)^2\omega$, then solve for ω.

(b) The kinetic energy of each skater is $K = \frac{1}{2}m(L/2)^2\omega^2$.

(c) The angular momentum of each skater is now $mR^2\omega$, where $R = 0.50\,\text{m}$. Equate $2mvL/2$ and $2mR^2\omega$, then solve for the new value of ω.

(d) Use $K = 2\frac{1}{2}mR^2\omega^2$, where $R = 0.50\,\text{m}$. The factor 2 arises because there are two skaters.

(e) Something did work on the skaters. What was it? Notice that while a skater moved along the pole she had a velocity with a component along the pole that was not zero.

The net external torque on the system consisting of the girl, the merry-go-round, and the rock is zero. The forces of the girl on the rock and the rock on the girl are internal forces. This means that the total angular momentum of the system is conserved when the girl throws the rock.

Before the rock is thrown the total angular momentum of the system is zero since the girl, merry-go-round, and rock are all at rest. Since it is conserved it is also zero after the rock is thrown. Then the angular momentum of the merry-go-round about its rotation axis is $I\omega$, where ω is its angular velocity. The angular momentum of the girl about the rotation axis is $MR^2\omega$, where M is the mass of the girl and R is the radius of the merry-go-round. The angular momentum of the rock about the rotation axis is mvR, where m is the mass of the rock and v is its velocity. Equate $I\omega + MR^2\omega + mvR$ to zero and solve for ω. The angular speed is the magnitude of ω. The linear speed of the girl is ωR.

(a) No external torques act on the system consisting of the disk and the cockroach, so the total angular momentum of the system is conserved as the cockroach walks from the rim of the disk to a point that is halfway to the center.

Initially the angular momentum of the disk about the rotation axis is $I\omega_0$, where I is its rotational inertia and ω_0 is its angular velocity. The angular momentum of the cockroach is $mR^2\omega_0$, where R is the radius of the disk and m is the mass of the cockroach.

After the cockroach reaches his final destination the angular momentum of the disk is $I\omega$, where ω is its angular velocity, and the angular momentum of the cockroach is $m(R/2)^2\omega$.

Equate the two expressions for the total angular momentum and solve for ω. According to Table 11–2 of the text the rotational inertia of the disk is given by $I = \frac{1}{2}(10.0m)R^2 = 5.00mR^2$. Substitute this expression for I. You should find that m and R cancel from the expression for ω and you should obtain $\omega = 1.14\omega_0$. Subtract ω_0 to find the change in the angular velocity of the disk.

(b) Initially the kinetic energy of the disk is $\frac{1}{2}I\omega_0^2$ and the kinetic energy of the cockroach is $\frac{1}{2}mR^2\omega_0^2$. Finally the kinetic energy of the disk is $\frac{1}{2}I\omega^2$ and the kinetic energy of the cockroach is $\frac{1}{2}m(R/2)^2\omega^2$. Substitute $I = 5.00mR^2$ and $\omega = 1.14\omega_0$.

If the rod does not swing significantly while the putty becomes attached, no external torques act on the rod-putty system during the collision and its angular momentum is conserved. The angular momentum before the collision is $I\omega_i$ where I is the rotational inertia of the rod about an end and ω_i is the angular speed of the rod before the collisions. After the collision the rod has angular momentum $I\omega_f$ and the putty has angular momentum $mL^2\omega$, where ω_f is the angular momentum of the rod (and putty) after the collision, L is the length of the rod, and m is the mass of the putty. Conservation of angular momentum yields

$$I\omega_i = (I + mL^{@})\omega_f \, .$$

Solve for ω_f.