

CHAPTER 16

HINT FOR EXERCISE 1

The object has zero velocity at the extreme points of its motion, so the time given is half a period and the distance given is twice the amplitude. Use $f = 1/T$ to calculate the frequency. The distance between the two points of zero velocity is half the amplitude.

[ans: (a) 0.50 s; (b) 2.0 Hz; (c) 18 cm]

(a) When a 15.0-kg object is hung from the balance, it extends 12.0 cm from its equilibrium position. Then, the force of gravity on the weight balances the force of the spring, so $kx = mg$ or $k = mg/x$. Here m is the mass of the object and x is the extension of the spring.

(b) The frequency of oscillation, in terms of the mass m of the package and the spring constant k , is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}.$$

Solve for m . The weight of the package is mg .

(a), (b), (c), and (d) The displacement is given by

$$x = (6.0 \text{ m}) \cos \left[(3\pi \text{ rad/s})t + \frac{\pi}{3} \text{ rad} \right],$$

the velocity is given by

$$v = \frac{dx}{dt} = -(18\pi \text{ m/s}) \sin[(3\pi \text{ rad/s})t + \pi/3 \text{ rad}],$$

and the acceleration is given by

$$a = \frac{dv}{dt} = -(54\pi^2 \text{ m/s}^2) \cos[(3\pi \text{ rad/s})t + \pi/3 \text{ rad}].$$

The phase is given by $(3\pi \text{ rad/s})t + \pi/3 \text{ rad}$. Evaluate these expressions for $t = 2.0 \text{ s}$. Don't forget to put your calculator in radian mode.

The angular frequency is $\omega = 3\pi \text{ rad/s}$. Use $\omega = 2\pi f$ to calculate the frequency and $T = 1/f$ to calculate the period.

(a) The spring force is given by $-kx$, where k is the spring constant and x is the extension or compression of the spring. This is the only force on the astronaut and the moving parts of the measuring device, considered as a single object. Thus Newton's second law is

$$-kx = (M + m) \frac{d^2x}{dt^2}.$$

Compare this equation with the equation for a single block of mass m on a spring. The comparison should tell you that the angular frequency of the astronaut is

$$\omega = \sqrt{\frac{k}{M + m}}.$$

The period T is related to the angular frequency by $\omega = 2\pi/T$. Use this expression to substitute for ω , then solve for M .

(b) Now $\omega = \sqrt{k/m}$.

(c) Substitute numerical values into the expression you found for ω in part (a).

(a) There are two forces acting on the block: the force of gravity, downward, and the force of the piston, upward. Since the block is not attached the piston cannot pull down on the block. The block is in simple harmonic motion as long as the piston is not required to pull downward to produce the motion.

Let N be the normal force of the piston on the block and m be the mass of the block. If up is taken to be positive, then Newton's second law for the block is

$$N - mg = ma.$$

If the block is in simple harmonic motion, then

$$a = -\omega^2 x_m \cos(\omega t + \phi)$$

and

$$N = mg - m\omega^2 x_m \cos(\omega t + \phi).$$

Since N cannot be negative $\omega^2 x_m$ must be less than g or x_m must be less than g/ω^2 . Use $\omega = 2\pi/T$ to calculate the angular frequency.

(b) The same analysis shows that ω must be less than $\sqrt{g/x_m}$ or the frequency must be less than $(1/2\pi)\sqrt{g/x_m}$.

Write

$$x = x_m \cos(\omega t + \phi)$$

for the coordinate of the block. Then its velocity is

$$v = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

and the acceleration is

$$a = \frac{dv}{dt} = -\omega^2 x_m \cos(\omega t + \phi).$$

Notice that $a/x = -\omega^2$. Use

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{-\frac{a}{x}}$$

to calculate the frequency. Use

$$\omega^2 = \frac{k}{m}$$

to calculate the mass of the block. Now, square the first two equations to obtain

$$x^2 + \left(\frac{v}{\omega}\right)^2 = x_m^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)] = x_m^2.$$

Thus

$$x_m = \sqrt{x^2 + \left(\frac{v}{\omega}\right)^2}.$$

[ans: (a) 5.58 Hz; (b) 0.325 kg; (c) 0.400 m]

Write

$$x_1 = x_m \sin(\omega t)$$

for the coordinate of one particle and

$$x_2 = x_m \sin(\omega t + \phi)$$

for the coordinate of the other. When they pass each other $x_1 = x_2 = x_m/2$. This means $\sin(\omega t) = 1/2$. Solve for ωt . There are many answers and you can use any one of them. It is easiest to use the smallest positive answer. Now solve

$$x_m/2 = \sin(\omega t + \phi)$$

for $\omega t + \phi$. Use the answers that are greater than ωt but less than $\omega t + 2\pi$ rad. Calculate ϕ for each of these.

The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \omega x_m \cos(\omega t)$$

and

$$v_2 = \frac{dx_2}{dt} = \omega x_m \cos(\omega t + \phi).$$

Try the values of ωt and $\omega t + \phi$ to see if they lead to velocities with different signs.

Suppose the block is displaced by x from its equilibrium position and calculate the force exerted on it by the two springs. If the net force is proportional to x and the constant of proportionality is negative, then the block will oscillate in simple harmonic motion and the square of its angular frequency will be the constant of proportionality divided by the mass of the block.

Suppose the block is at x_1 when the force of spring 1 on it vanishes and suppose it is at x_2 when the force of spring 2 on it vanishes. Put the origin at the position of the block when the net force on it (due to the two springs together) is zero. Then, $k_1x_1 + k_2x_2 = 0$. Now, displace the block to x . The force of spring 1 on it is $-k_1(x - x_1)$ and the force of spring 2 on it is $-k_2(x - x_2)$. The net force is

$$F = -k_1(x - x_1) - k_2(x - x_2) = -(k_1 + k_2)x,$$

where the relation between x_1 and x_2 , derived above, was used. The angular frequency is therefore

$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

or, since $\omega_1 = \sqrt{k_1/m}$ and $\omega_2 = \sqrt{k_2/m}$,

$$\omega = \sqrt{\omega_1^2 + \omega_2^2}.$$

Divide by 2π to obtain the desired result.

Three forces act on the block: the force of the spring, the force of gravity, and the normal force of the incline. When the block is at rest these forces sum to zero. Take a coordinate axis to be parallel to the incline with the positive direction down the incline. The component of the gravitational force along that axis is $mg \sin \theta$, where m is the mass of the block and θ is the angle that the incline makes with the horizontal. The component of the spring force is $-kx$, where k is the spring constant and x is the extension of the spring from its relaxed length. The component of the normal force is zero. Thus the condition that the net force be zero yields $mg \sin \theta - kx = 0$. Solve for x and add the result to the relaxed length of the spring to find the distance of the block from the top of the incline.

When the block is displaced and released it accelerates. Newton's second law gives

$$mg \sin \theta - kx = m \frac{d^2 x}{dt^2} .$$

Try the solution

$$x = (mg/k) \sin \theta + A \sin(\omega t)$$

and show that it satisfies Newton's second law provided the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} .$$

The period is

$$T = 2\pi \sqrt{\frac{m}{k}} .$$

(a) Use

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}.$$

(b) and (c) Use

$$U = \frac{1}{2} kx^2$$

to calculate the potential energy and

$$K = \frac{1}{2} mv^2$$

to calculate the kinetic energy. The mechanical energy is the sum of the two. When the block reaches x_m , the potential energy is $\frac{1}{2} kx_m^2$ and the kinetic energy is 0. Set $E_{\text{mec}} = \frac{1}{2} kx_m^2$ and solve for x_m .

[ans: (a) 2.25 Hz; (b) 125 J; (c) 250 J; (d) 86.6 cm]

(a) According to Table 11–2 the rotational inertia of a uniform horizontal disk about a vertical axis through its center is given by $I = \frac{1}{2}MR^2$, where M is its mass and R is its radius.

(b) The torsion constant κ is the constant proportionality between the torque and the angular displacement from the equilibrium orientation. Thus

$$\kappa = \frac{\tau}{\theta},$$

where τ ($= 0.600 \text{ N} \cdot \text{m}$) is the torque and θ ($= 2.50 \text{ rad}$) is the angular displacement.

(c) When the disk is set into oscillation the angular frequency is given by

$$\omega = \sqrt{\frac{\kappa}{I}}.$$

The period is given by

$$T = 2\pi\sqrt{\frac{I}{mgd}},$$

where I is the rotational inertia of the disk about the pivot point. Use the parallel axis theorem to find an expression for I . Since the rotational inertia of a disk about an axis through its center of mass is $I_{\text{com}} = \frac{1}{2}MR^2$ (see Table 11-2 of the text) and the center of mass is a distance d from the pivot, $I = \frac{1}{2}MR^2 + Md^2$.

(a) Use

$$T = 2\pi\sqrt{\frac{I}{mgx}}.$$

Use the parallel axis theorem to find an expression for the rotational inertia I . The rotational inertia of the stick about an axis through its center of mass is $I_{\text{com}} = ML^2/12$ (see Table 11-2 of the text). The pivot is a distance x from the center of mass, so

$$I = \frac{ML^2}{12} + Mx^2.$$

(b) Set the derivative of T with respect to x equal to 0 and solve for x/L . Easier yet, solve $dT^2/dx = 0$ for x/L .

(c) Substitute $x/L = \sqrt{1/12}$ into the expression you found for T .

(a) If the wheel rotates through a small angle θ the spring elongates by very nearly $r\theta$, for θ in radians. The force exerted by the spring has a magnitude of $F = kr\theta$, where k is the spring constant. The torque exerted by the spring is $\tau = Fr = kr^2\theta$. Thus the torque is proportional to the angular displacement and the constant of proportionality is $\kappa = kr^2$. Use

$$\omega = \sqrt{\frac{\kappa}{I}},$$

where I is the rotational inertia of the wheel, to find the angular frequency ω . The rotational inertia of the wheel is $I = mR^2$.

(b) Put r equal to R in the expression you found for T in part (a).

(c) Put r equal to zero in the expression for T .

(a) The spring compresses by $x = 0.10$ m when the load is $m = 500$ kg. Then the spring and gravitational forces sum to zero, so $mg = kx$. Solve for the spring constant k .

The coordinate x of an object in damped harmonic motion is given by

$$x = x_m e^{-bt/2m} \sin(\omega t)$$

where m is its mass, t is the time, b is the damping coefficient, and ω is the angular frequency. The problem statement tells us that when $t = T$, where T is the period, then

$$x_m e^{-bT/2m} = \frac{1}{2} x_m.$$

Solve for b . You should get

$$b = \frac{2m}{T} \ln 2.$$

The angular frequency is nearly $\sqrt{k/m}$, so the period is nearly $2\pi\sqrt{m/k}$.

When the car is stationary, the force of the suspension balances the force of gravity: $W = kx$, where W is the weight of the car and any passengers, k is the spring constant of the suspension, and x is the compression of the suspension. When the weight changes by ΔW as the passengers leave the car, the car rises by $\Delta x = \Delta W/k$. You need to find the spring constant k . The time interval between bumps is given by $T = d/v$, where d is the distance between the corrugations and v is the speed of the car. Since the oscillation of the car has maximum amplitude we take this to be the resonance period of the suspension. The resonance angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{d}.$$

Use $k = m\omega^2$, where m is the mass of the car and passengers, to find k .