Chapter 17 Hint for Exercise 4

Use $\omega = 2\pi f$ to find the angular frequency and $v = \omega/k$ to find the angular wave number. The position and time dependent terms in the expression for the displacement must have the same sign since the wave is traveling in the negative x direction. Take the displacement to be parallel to the y axis.

(a) through (f) The amplitude is 6.0 cm, the angular wave number is $k = 0.020\pi\,\text{rad/m}$, and the angular frequency is $\omega = 4.0\pi\,\text{rad/s}$. Use $k = 2\pi/\lambda$ to find the wavelength, $\omega = 2\pi f$ to find the frequency, and $v = \lambda f$ to find the wave speed. Look at the signs of the two terms in the argument of the sine function to determine the direction of travel. The transverse speed of the point at x, at time t, is given by

$$u(x,t) = \frac{\partial y}{\partial t} = (6.0\,\text{cm})4.0\pi\cos[(0.020\,\text{rad/m})\pi x + (4.0\,\text{rad/s})\pi t]$$

and its maximum value is

$$u_m = (6.0\,\text{cm})(4.0\,\text{s}^{-1})\pi\,.$$

Evaluate this expression.

(g) Evaluate

$$y = (6.0\,\text{cm})\sin[(0.020\,\text{rad/m})\pi x + (4.0\,\text{rad/s})\pi t]$$

for the given values of the coordinate and time.

(b) Read the period T from the graph, then use $v = \lambda/T$ to find the wave speed. Here λ is the wavelength, which is given.

(c) Write

$$y = y_m \sin(kx - \omega t + \phi)\,,$$

where y_m is the amplitude, k is the angular wave number, ω is the angular frequency, and ϕ is a phase constant. Notice that a minus sign was used because the wave is moving in the positive x direction. The amplitude can be read from the graph the angular wave number is given by $k = 2\pi/\lambda$, and the angular frequency is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency. You can find the phase constant by observing that $y = 0$ when $x = 0$ and $t = 0$ and that the string velocity is positive at $x = 0$ and $t = 0$. The first condition tells us that $\sin\phi = 0$, so ϕ is either zero or π rad. The second condition tells us that $-\cos\phi$ is positive.

(d) Evaluate

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t + \phi)\,.$$

Chapter 17 Hint for Problem 9

(a) The phase is given by $kx \pm \omega t$, where k is the angular wave number and ω is the angular frequency. The difference in phase of two points separated by Δx is $k\,\Delta x$. Set this expression equal to $\pi/3$ and solve for Δx. You must first find the value for the angular wave number k. Use $v = \lambda f$ to find the wavelength λ, then use $k = 2\pi/\lambda$ to find k.

(b) Now, $\Delta x = 0$ and the phase difference is given by $\omega\,\Delta t$. Evaluate this expression. The angular frequency is given by $\omega = 2\pi f$.

[ans: (a) 0.117 m; (b) 3.14 rad]

Chapter 17 Hint for Exercise 14

(a) Use $v = \omega/k$ to find the wave speed v. Here ω is the angular frequency and k is the angular wave number. Remember that the displacement in a wave is written

$$y = y_m \sin(kx - \omega t + \phi)\,.$$

Compare this form with the equation given in the exercise statement to find the numerical values of k and ω.

(b) Solve

$$v = \sqrt{\frac{\tau}{\mu}}$$

for the linear mass density μ. Here τ is the tension in the string.

The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}}\,,$$

where τ is the tension in the wire and μ is the linear density. The stress S is the force per unit area on a cross section of the wire, so the tension is given by

$$\tau = \pi R^2 S\,,$$

where R is the radius, and the linear density is the density times the cross-sectional area:

$$\mu = \pi R^2 \rho\,.$$

Substitute these expressions into the equation for the speed. Your answer should not depend on R. Evaluate the result.

(a) and (b) First find the tension in each string. Because the pulley is in rotational equilibrium the net torque on it is zero. If τ_1 is the tension in string 1 and τ_2 is the tension in string 2, then $(\tau_1 - \tau_2)R = 0$ and $\tau_1 = \tau_2$. Since the pulley is in translational equilibrium the net force on it is zero: $Mg - \tau_1 - \tau_2 = 0$. The solution to these equations is $\tau_1 = Mg/2$ and $\tau_2 = Mg/2$. Substitute into

$$v_1 = \sqrt{\frac{\tau_1}{\mu_1}}$$

and

$$v_2 = \sqrt{\frac{\tau_2}{\mu_2}}\,.$$

(c) and (d) The tension in string 1 is $M_1 g$ and the wave speed is

$$v_1 = \sqrt{\frac{M_1 g}{\mu_1}}\,.$$

The tension in string 2 is $M_2 g$ and the wave speed is

$$v_2 = \sqrt{\frac{M_2 g}{\mu_2}}\,.$$

Since the wave speeds are to be equal, $M_1/\mu_1 = M_2/\mu_2$. Solve this simultaneously with $M_1 + M_2 = M$ for M_1 and M_2.

(a) The wave speed is given by

$$v = \sqrt{\frac{F}{mu}}\,,$$

where μ is the linear density. Use

$$F = k\,\Delta\ell$$

and

$$\mu = \frac{m}{\ell + \Delta\ell}$$

to write v in terms of m, k, ℓ and $\Delta\ell$.

(b) The time is given by

$$t = \frac{\ell + \Delta\ell}{v} = \frac{\ell + \Delta\ell}{\sqrt{k\,\Delta\ell(\ell + \Delta\ell)/m}}\,.$$

If $\Delta\ell \ll \ell$, then $\ell + \Delta\ell$ may be replaced by ℓ. You should obtain

$$t \approx \sqrt{\frac{\ell m}{k\,\Delta\ell}}\,.$$

If $\Delta\ell \gg \ell$, then you may replace $\ell + \Delta\ell$ with $\Delta\ell$. You should get

$$t \approx \sqrt{\frac{m}{k}}\,.$$

Chapter 17 Hint for Exercise 24

The average power is given by

$$P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2\,,$$

where μ is the linear density, v is the wave speed, ω is the angular frequency, and y_m is the amplitude. Substitute $\mu = m/L$, $v = \sqrt{\tau/\mu}$, and $\omega = 2\pi f$, where m is the mass of the string, L is its length, τ is the tension in the string, and f is the frequency of the wave. Solve for f.

If the two waves are

$$y_1 = y_m \sin(kx - \omega t + \phi)$$

and

$$y_2 = y_m \sin(kx - \omega t)\,,$$

then

$$y_1 + y_2 = 2y_m \sin(kx - \omega t + \phi/2)\cos(\phi/2)\,.$$

The amplitude is $2y_m \cos(\phi/2)$ ad you want this to be $1.50y_m$. Solve for $\cos(\phi/2)$, then ϕ. In terms of the wavelength λ the phase difference can be written $(\phi/2\pi)\lambda$, where ϕ is in radians.

(a) The angular wave number k is $20\,\mathrm{m}^{-1}$ and is related to the wavelength λ by

$$k = \frac{2\pi}{\lambda}\,.$$

(b) The phase constant of the resultant wave is half the phase difference of the two constituent waves.

(c) The amplitude of the resultant wave is $y_m = 3.0\,\mathrm{mm}$ and is related to the amplitude y_{1m} of either of the constituent waves by

$$y_m = 2y_{1m}\cos(\phi/2)\,,$$

where ϕ is the phase difference of the constituent waves.

Write

$$y = y_{1m}\sin(kx - \omega t) + y_{2m}\sin(kx - \omega t + \phi)\,,$$

where y_{1m} is the amplitude of the first wave, y_{2m} is the amplitude of the second wave, k is the angular wave number of both waves, ω is the angular frequency of both waves, and ϕ is the phase constant for the second wave. Draw a phasor diagram and use the law of cosines to show that the amplitude of the resultant wave is

$$y_m^2 = y_{1m}^2 + y_{2m}^2 + 2y_{1m}y_{2m}\cos\phi\,.$$

Solve for ϕ.

The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}}\,,$$

where τ is the tension in the string and μ is the linear density of the string. According to the diagram there are three half-wavelengths in the length of the string, so $L = 3\lambda/2$, where λ is the wavelength. Solve for λ. Finally, use $v = f\lambda$ to find the frequency f.

[ans: (a) 140 m/s; (b) 60 cm; (c) 240 Hz]

HINT FOR EXERCISE 38

The resonant frequencies of string B are given by

$$f_{Bn} = \frac{nv}{8L}\,,$$

where v is the wave speed and n is a positive integer. The resonant frequencies of string A are given by

$$f_{Am} = \frac{v}{2L}\,,$$

where m is an integer. Find values of n and m so that the frequencies match.

Chapter 17 Hint for Problem 42

(a) Write

$$y = (0.050)\left[\cos(\pi x - 4\pi t) + \cos(\pi x + 4\pi t)\right] .$$

Use the trigonometric identity

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) ,$$

which can be found in Appendix E of the text. Let $\alpha = \pi x - 4\pi t$ and $\beta = \pi x + 4\pi t$ and show that

$$y = 2(0.050)\cos(\pi x)\cos(4\pi t) .$$

Nodes occur for $\pi x = n\pi/2$, where n is an integer. You want the smallest possible positive value for x.

(b) The string velocity is

$$u = \frac{\partial y}{\partial t} = -8\pi(0.050)\cos(\pi x)\sin(4\pi t)$$

and at $x = 0$

$$u = -8\pi(0.050)\sin(4\pi t) .$$

The velocity is zero for $4\pi t = n\pi$, where n is zero or an integer. You want all the values of t such that $0 \geq t \geq 5.0\,\mathrm{s}$.

(a) For the second harmonic the length of the rope is equal to one wavelength. You can find the wavelength by observing that the angular wave number is $k = \pi/2$ and the wavelength is $\lambda = k/2\pi$.

(b) The wave speed v is given by

$$v = \frac{\omega}{k},$$

where ω is the angular frequency. For this wave it is $\omega = 12\pi$.

(c) Use

$$v = \sqrt{\frac{\tau}{\mu}},$$

where τ is the tension in the rope and μ is the linear mass density of the rope. Substitute $\mu = M/L$, where M is the mass of the rope and L is its length, then solve for M.

(d) An integer number of half wavelengths must fit along the rope, so possible values of the wavelength are

$$\lambda_n = \frac{2L}{n},$$

where n is an integer. The possible periods are

$$T_n = \frac{\lambda_n}{v} = \frac{2L}{nv}.$$

For the third harmonic $n = 3$. Calculate T_3.