

CHAPTER 20 HINT FOR EXERCISE 6

Use the ideal gas law $pV = NkT$, where p is the pressure, V is the volume, N is the number of molecules, T is the temperature in kelvins, and k is the Boltzmann constant (1.3807×10^{-23} J/K). Solve for N

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The work done by the external agent is the negative of the work done by the gas and is given by

$$W_{\text{ext}} = - \int_{V_1}^{V_2} p \, dV,$$

where p is the pressure, V_1 is the initial volume, and V_2 is the final volume. Now $p = nRT/V$, where n is the number of moles and T is the temperature in kelvins. Furthermore, the compression is isothermal, so T is constant. Thus

$$W_{\text{ext}} = -nRT \int_{V_1}^{V_2} \frac{dV}{V} = -nRT \ln \frac{V_2}{V_1}.$$

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The partial pressure due to gas 1 is given by $p_1 = n_1RT/V$, where n_1 is the number of moles of that gas, R is the gas constant (8.315 J/mol·K), T is the temperature, and V is the volume of the container. Similarly, the partial pressure due to gas 2 is given by $p_2 = n_2RT/V$. Note that T and V each have the same values in the two expressions for the partial pressures. The total pressure is $p = p_1 + p_2$ and the fraction that is attributable to gas 2 is $p_2/(p_1 + p_2)$.

- (a) Use the ideal gas law $pV = nRT$, where p is the pressure, V is the volume, n is the number of moles of gas, R ($= 8.315 \text{ J/mol} \cdot \text{K}$) is the gas constant, and T is the temperature in kelvins. Solve for n .
- (b) Now solve $pV = nRT$ for T .
- (c) Again solve $pV = nRT$ for T or better yet consider the ratio

$$\frac{T_c}{T_a} = \frac{V_c}{V_a}.$$

This was obtained by dividing the ideal gas equation for point c by the ideal gas equation for point a.

- (d) The work done by the gas is the area enclosed by the graph. Since the graph is a triangle its area is half the product of the base (2.0 m^3) and the altitude (5.0 kPa). Since the process is a cycle, the change in the internal energy is zero and, according to the first law of thermodynamics, the work done by the gas is equal to the energy absorbed by the gas as heat.

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The air in the tube was originally at atmospheric pressure and occupied the full length L of the tube. It was compressed isothermally to a length of $L/2$. That is, its volume was halved. The ideal gas law ($pV = nRT$) tells us that when the volume is isothermally halved the pressure is doubled. Thus the pressure after compression is $p = 2p_0$, where p_0 is atmospheric pressure.

Now use what you learned in Chapter 15 about incompressible fluids, the water in this case. The pressure increases with depth: the pressure at a point in the fluid that is a distance h below the surface is ρgh greater than the pressure at the surface. Here ρ is the density of the fluid. Consider a point at the bottom of the tube. The pressure there is greater than the pressure at the interface with the air in the tube by $\rho gL/2$. That is, it is

$$p_b = p + \rho gL/2.$$

On the other hand it is ρgh greater than atmospheric pressure. That is, it is

$$p_b = p_0 + \rho gh.$$

Equate the two expressions for p_b and solve for h .

(a) Evaluate

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where M is the molar mass of N_2 (14 g/mol). Be sure to change the temperature value to the Kelvin scale.

(b) and (c) Solve

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

for T .

(a) Since the total mass of the gas is nM , where n is the number of moles and M is the molar mass, the density is given by $\rho = nM/V$, where V is the volume. The pressure is given by

$$p = \frac{1}{3}\rho v_{\text{rms}}^2.$$

Solve for v_{rms} . Recall that $1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$.

(b) The average kinetic energy per mole is given by $3RT/2$ and by $Mv_{\text{rms}}^2/2$. Equate these two expressions to obtain

$$M = \frac{3RT}{v_{\text{rms}}^2}.$$

Evaluate this expression.

(a) Use $\rho = nM/V$ to obtain $n = \rho V/M$. Substitute this expression for n in the ideal gas law $pV = nRT$.

(b) Substitute $R = N_A k$ into $pV = nRT$, then replace $N_A n$ with N .

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According to Eq. 20-25 the mean free path of a molecule in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of the molecule and N/V is the number of molecules per unit volume.

(a) Assume both argon and nitrogen gases obey the ideal gas law ($pV = NkT$, where p is the pressure, k is the Boltzmann constant, and T is the temperature). Since the two gases are at the same pressure and are at the same volume, N/V is the same for them. This means that

$$\frac{\lambda_A}{\lambda_N} = \frac{d_N^2}{d_A^2},$$

where the subscript A refers to argon and the subscript N refers to nitrogen. Solve for d_A/d_N .

(b) Use the ideal gas law to show that $N/V = p/RT$ and therefore that

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 p}.$$

Here p is the pressure, T is the temperature, and R is the gas constant (8.315 J/mol · K). The ratio of the mean free paths for the same gas at the same temperature but two different pressures is

$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1}.$$

(c) The ratio of the mean free paths for the same gas at the same pressure but two different temperatures is

$$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2}.$$

Don't forget that the temperature must be in kelvins.

According to Eqs. 20–31 and 20–34 the average speed v_{avg} and rms speed v_{rms} of a gas of molecules with molar mass M , at temperature T , are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$$

and

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where R is the gas constant (8.315 J/mol · K). Set the expression for the average speed of the molecules in gas 2 equal to twice the rms speed of the molecules in gas 1, then solve for the mass ratio. The ratio of the molar masses is the same as the ratio of the atomic masses.

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Take the internal energy to be 0 at temperature $T = 0$ K. Then, the internal energy at any temperature is given by

$$E_{\text{int}} = nC_V T ,$$

where C_V is the molar specific heat for a constant volume process. For an ideal monatomic gas $C_V = \frac{3}{2}R$.

(a) Use the first law of thermodynamics

$$\Delta E_{\text{int}} = Q - W ,$$

where ΔE_{int} is the internal energy, W is the work done by the gas, and Q is the energy taken in by the gas in the form of heat. Since the pressure is constant the work W done by the gas is $p \Delta V$. Since the volume increases W is positive.

(b) Since $pV = nRT$ the change in volume and change in temperature are related by $p \Delta V = nR \Delta T$, so

$$\Delta T = \frac{p}{nR} \Delta V .$$

The molar specific heat is determined by

$$C_p = \frac{Q}{n \Delta T} = \frac{R}{p} \frac{Q}{\Delta V} .$$

(c) Use

$$C_V = \frac{\Delta E_{\text{int}}}{n \Delta T} = \frac{R}{p} \frac{\Delta E_{\text{int}}}{\Delta V} .$$

(a) The change in the internal energy of an ideal gas during a process is given by

$$\Delta E_{\text{int}} = nC_V \Delta T,$$

where n is the number of moles, C_V is the molar specific heat, and ΔT is the change in temperature. The molar specific heat of an ideal diatomic gas is $C_V = (5/2)R$, where R ($= 8.315 \text{ J/mol} \cdot \text{K}$) is the gas constant. You need to find the temperatures at points a and c . Solve the ideal gas law $p_a V_a = nRT_a$ for T_a and the ideal gas law $p_c V_c = nRT_c$ for T_c .

(b) Use the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q - W,$$

where Q is the energy taken in by the gas in the form of heat and W is the work done by the gas. You can calculate W and solve the first law equation for Q . The work done by the gas is given by

$$W = \int_{V_a}^{V_c} p \, dV.$$

You might use the graph to find the pressure p as a function of the volume V . It is a linear so you can write $p = A + BV$, where A and B are constants. Choose the constants so $p = 5.0 \text{ kPa}$ when $V = 2.0 \text{ m}^3$ and $p = 2.0 \text{ kPa}$ when $V = 4.0 \text{ m}^3$. Then substitute the function into the integrand and evaluate the integral. The change in the internal energy is the same as for the diagonal path since the internal energy is a state function.

Alternatively, you might recognize that the integral for the work is the area under the line from a to c . This area can be divided into two pieces: a triangle, whose area is half the product of its base and altitude, and a rectangle, whose area is the product of two perpendicular sides.

(c) Again use the first law of thermodynamics. The gas does work only during the portion of the process from a to b and this work is $W = p_a(V_b - V_a)$.

(a) According to Table 20–1 of the text the molar mass of oxygen is $M = 32.0$ g/mol. If m is the mass of the sample, then the number of moles present is $n = m/M$.

(b) Use

$$Q = nC_p \Delta T.$$

Since the molecules are diatomic $C_p = C_V + R = (5/2)R + R = 7R/2$.

(c) The change in the internal energy is given by

$$\Delta E_{\text{int}} = nC_V \Delta T,$$

where $C_V = 5R/2$ is the molar heat capacity for a constant volume process. The fraction of the energy transferred as heat that is used to increase the internal energy is given by

$$f = \frac{\Delta E_{\text{int}}}{Q}.$$

(a) For an adiabatic change in volume of an ideal gas from volume V_1 to volume V_2

$$p_1 V_1^\gamma = p_2 V_2^\gamma ,$$

where p_1 is the pressure when the volume is V_1 and p_2 is the pressure when the volume is V_2 . Let $p_1 = 1.0 \text{ atm}$ and $v_2 = V_1/2$, then solve for p_2 . The ideal gas law tells us that $p_1 V_1 = nRT_1$ and $p_2 V_2 = nRT_2$, where T_1 is the temperature when the volume is V_1 and T_2 is the temperature when the volume is V_2 . Both temperatures are in kelvins. Use the first equation to eliminate n from the second and obtain

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 .$$

(b) Let V_3 be the final volume, P_3 be the final pressure, and T_3 be the final temperature. Then $p_3 V_3 = nRT_3$. Similarly, $p_2 V_2 = nRT_2$. Since $p_3 = p_2$, these equations lead to $V_3/T_3 = V_2/T_2$. Solve for V_3 .

(a) Let p_0 , V_0 , and T_0 be the initial pressure, volume, and temperature, respectively. Let p_1 , V_1 , and T_1 be the final pressure, volume, and temperature respectively. The work done by the gas and the energy input to it as heat are both zero during the process. This means the internal energy does not change and, since the internal energy of an ideal gas depends only on the temperature, the temperature does not change. Thus $T_1 = T_0$. According to the ideal gas law $p_1V_1 = nRT_1$ and $p_0V_0 = nRT_0$. Solve these equations for T_1 and T_0 , then equate the two expressions to each other and solve for p_1 . You should get $p_1 = p_0V_0/V_1$. Set $V_1 = 3.00V_0$.

(b) Let p_2 and V_2 be the final pressure and volume. The process is adiabatic, so you can use the relationship

$$p_1V_1^\gamma = p_2V_2^\gamma.$$

Substitute $V_2 = V_1/3.00$, $p_2 = (3.00)^{1/3}p_0$, and $p_1 = p_0/3$. Solve for γ . Recall that $\gamma = 5/3$ for a monatomic ideal gas, $\gamma = 7/5$ for a diatomic ideal gas, and $\gamma = 4/3$ for a polyatomic ideal gas.

(c) The average kinetic energy per molecule is proportional to the temperature in kelvins. Use the ideal gas law to show that $T_2/T_0 = p_2/p_0$. Thus $K_2/K_0 = p_2/p_0$.

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(a) For an adiabatic process

$$p_1 V_1^\gamma = p_2 V_2^\gamma .$$

The solution for γ is

$$\gamma = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} .$$

Evaluate this expression and compare your result with 1.67 (monatomic), 1.40 (diatomic), and 1.29 (polyatomic).

(b) Use the ideal gas law in the form

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} .$$

Solve for T_2 .

(c) Use the ideal gas law in the form

$$p_1 V_1 = nRT_1 .$$

Solve for n .

(d) The translational kinetic energy per mole is given by $\frac{3}{2}RT$. For the energy before compression substitute $T_1 = 273\text{ K}$ and for the energy after compression substitute $T_2 = 2.73 \times 10^4\text{ K}$.

(e) Since the kinetic energy is given by

$$K = \frac{1}{2} M v_{\text{rms}}^2$$

the ratio of the rms speeds is the same as the ratio of the kinetic energies.

[ans: (a) $\gamma = 1.67$, monatomic; (b) $2.7 \times 10^4\text{ K}$; (c) $4.5 \times 10^4\text{ mol}$; (d) $3.40 \times 10^3\text{ J}$, $3.40 \times 10^5\text{ J}$; (e) 0.010]

For each case let p_i , V_i , and T_i be the initial pressure, volume, and temperature respectively and let p_f , V_f , and T_f be the final pressure, volume, and temperature.

(a) The process is isothermal, so $T_f = T_i$. Use the ideal gas law to show that $p_f V_f = p_i V_i$. Solve for p_f . The work done by the gas is

$$W = \int_{V_i}^{V_f} p \, dV = nRT_i \int_{V_i}^{V_f} \frac{dV}{V} = nRT_i \ln \left(\frac{V_f}{V_i} \right) = p_i V_i \ln \left(\frac{V_f}{V_i} \right).$$

The ideal gas law was used to substitute for p in the integrand and in the last step it was used to substitute for nRT_i .

(b) The process is adiabatic, so $p_i V_i^\gamma = p_f V_f^\gamma$, with $\gamma = 5/3$. Solve for p_f . Use the ideal gas law to find the final temperature: solve $p_f V_f = nRT_f$ for T_f . The energy taken in by the gas as heat is zero, so the work done by the gas is the negative of the change in the internal energy. Thus

$$W = -\Delta E_{\text{int}} = nC_V \Delta T,$$

where $C_V (= 3R/2)$ is the molar specific heat. Thus $W = -(3/2)nR(T_f - T_i)$.

(c) This part is worked the same way as part (b) but with $\gamma = 7/5$ and $C_V = (5/2)R$.