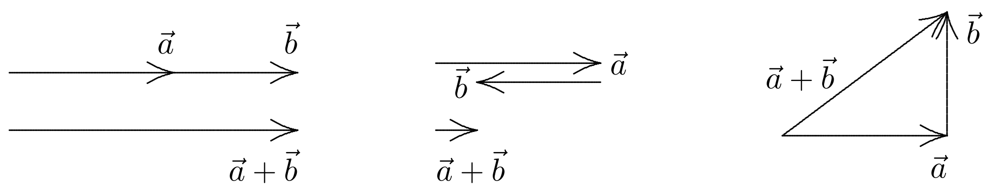
Chapter 3 Hint for Exercise 1

Draw the 4-m long displacement vector. Now, imagine the 3-m long displacement vector drawn with its tail at the head of the longer vector. Rotate it about its tail and for each orientation check the magnitude of the vector sum, a vector from the tail of the 4-m vector to the head of the 3-m vector. Observe the orientation when the vector sum has each of the desired magnitudes.

The magnitude of the vector sum is 7 m when the vectors are in the same direction. It has a magnitude of 1 m when the vectors are in opposite directions. It has a magnitude of 5 m when the vectors make a right angle with each other. The orientations are shown in the diagrams below.



Chapter 3 Hint for Problem 8

(a) The magnitude of a vector is the square root of the sum of the squares of its components. In this case, the vector is denoted by AB on Fig. 3–28 and its components are $\overline{\text{AC}}$ and $\overline{\text{AD}}$. Notice that $\overline{\text{AC}}$ and $\overline{\text{AD}}$ are perpendicular to each other.

(b) Evaluate $\overline{\text{AB}} \sin 52.0°$.

CHAPTER 3 HINT FOR EXERCISE 12

(b) and (c) You want to find the magnitude and direction of the vector sum of the three displacement vectors that represent the person's walking pattern. Take the x axis to run west to east and the y axis to run south to north. Then the three displacement vectors are

$$\vec{r}_1 = (3.1\,\text{km})\,\hat{\imath}\,,$$

$$\vec{r}_2 = -(2.4\,\text{km})\,\hat{\jmath}\,,$$

and

$$\vec{r}_3 = -(5.2\,\text{m})\,\hat{\jmath}\,.$$

Find the sum

$$\vec{r} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$$

in unit vector notation. The magnitude is given by

$$r = \sqrt{r_x^2 + r_y^2}$$

and the angle θ that $\vec{r}$ makes with the positive x axis is given by

$$\tan\theta = \frac{r_y}{r_x}\,.$$

You may need to add 180° to the angle your calculator gives you. Look at the sketch you made in part (a) and be sure that the value you accept for θ is consistent with it.

(a) and (b) Use

$$a = \sqrt{a_x^2 + a_y^2}$$

and

$$\tan\theta = \frac{a_y}{a_x}\,.$$

Notice that the x component of $\vec{a}$ is positive and the y component is negative. This means that θ is between 270° and 360° if θ is measured counterclockwise from the positive x direction. You may also give the result as a negative angle between 0 and −90°.

(c) and (d) Use

$$b = \sqrt{b_x^2 + b_y^2}$$

and

$$\tan\theta = \frac{b_y}{b_x}\,.$$

Notice that both components $\vec{b}$ are positive. This means that θ is between 0 and 90° if θ is measured counterclockwise from the positive x direction.

(e) and (f) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x$ and $c_y = a_y + b_y$. Use

$$c = \sqrt{c_x^2 + c_y^2}$$

and

$$\tan\theta = \frac{c_y}{c_x}\,.$$

Both components of $\vec{c}$ are positive, so θ is between 0 and 90°.

(g) and (h) Let $\vec{c} = \vec{b} - \vec{a}$. Then $c_x = b_x - a_x$ and $c_y = b_y - a_y$. Use

$$c = \sqrt{c_x^2 + c_y^2}$$

and

$$\tan\theta = \frac{c_y}{c_x}\,.$$

Both components of $\vec{c}$ are positive, so θ is between 0 and 90°.

(i), (j), and (k) The vector $\vec{a} - \vec{b}$ has the same magnitude as the vector $\vec{b} - \vec{a}$ and is in the opposite direction. Add 180° to the angle for $\vec{b} - \vec{a}$.

(k) Remember that $\vec{b} - \vec{a}$ and $\vec{a} - \vec{b}$ are in opposite directions.

Chapter 3 Hint for Problem 22

Solve $\vec{A} + \vec{B} = \vec{C}$ for $\vec{B}$:

$$\vec{B} = \vec{C} - \vec{A}\,.$$

Find the components of $\vec{C}$ and $\vec{A}$, then use

$$B_x = C_x - A_x$$

and

$$B_y = C_y - A_y$$

to find the components of $\vec{B}$. The magnitude of $\vec{B}$ is given by

$$B = \sqrt{B_x^2 + B_y^2}$$

and the tangent of the angle θ that $\vec{B}$ makes with the positive x axis is given by

$$\tan\theta = \frac{B_y}{B_x}\,.$$

Remember that there are two solutions for θ and that they differ by 180°. Carefully note the signs of the components of $\vec{B}$ and choose the value of θ that indicates a vector in the correct direction.

CHAPTER 3 HINT FOR EXERCISE 28

(a) The vector $\vec{A}$ makes an angle of $\theta' = 60^\circ - 20^\circ = 40^\circ$ with the positive x' axis. Use

$$A_{x'} = A\cos\theta'$$

and

$$A_{y'} = A\sin\theta'$$

to compute the components of $\vec{A}$ in the primed coordinate system.

(b) Use

$$B = \sqrt{B_x^2 + B_y^2}$$

and

$$\theta = \tan^{-1}\frac{B_y}{B_x}$$

to find the magnitude of $\vec{B}$ and the angle it makes with the positive x axis. Subtract 20° from θ to find the angle that $\vec{B}$ makes with the x' axis, then use

$$B_{x'} = B\cos\theta'$$

and

$$B_{y'} = B\sin\theta'$$

to find the components in the primed coordinate system.

Chapter 3 Hint for Problem 34

Let $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$, then evaluate the vector product. You should get

$$F_x\,\hat{\imath} + F_y\,\hat{\jmath} + F_z\,\hat{k} = q(v_yB_z - v_zB_y)\,\hat{\imath} + q(v_zB_x - v_xB_z)\,\hat{\jmath} + q(v_xB_y - v_yB_x)\,\hat{k}\,.$$

This is equivalent to the three equations

$$F_x = q(v_yB_z - v_zB_y)\,,$$

$$F_y = q(v_zB_x - v_xB_z)\,,$$

$$F_z = q(v_xB_y - v_yB_x)\,.$$

Replace B_y with B_x and solve the resulting equations simultaneously for B_x and B_z. First solve the third equation for B_x, then substitute its value into either of the other two equations and solve for B_z.

(a) The scalar product is given by

$$\vec{a} \cdot \vec{b} = ab \sin \phi \,,$$

where ϕ is the angle between $\vec{a}$ and $\vec{b}$ when the two vectors are drawn with their tails at the same point. In terms of components it is given by

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \,.$$

Thus

$$\cos \phi = \frac{a_x b_x + a_y b_y}{ab} \,.$$

Evaluate this quantity, then find the inverse cosine. Use

$$a = \sqrt{a_x^2 + a_y^2}$$

and

$$b = \sqrt{b_x^2 + b_y^2}$$

to find the magnitudes of the vectors.

(b), (c), (d), and (e) Since $\vec{c}$ is perpendicular to $\vec{a}$,

$$\vec{a} \cdot \vec{c} = 0 \,.$$

Since $\vec{c}$ is in the xy plane, this means

$$a_x c_x + a_y c_y = 0 \,.$$

Now

$$c = \sqrt{c_x^2 + c_y^2} \,,$$

so

$$c_y = \sqrt{c^2 - c_x^2} \,.$$

Solve

$$a_x c_x + a_y \sqrt{c^2 - c_x^2} = 0$$

for c_x. There are two solutions. You want the positive one. The negative one is actually d_x. Use

$$a_x c_x + a_y c_y = 0$$

to find c_y and a similar equation to find d_y.