CHAPTER 6 HINT FOR EXERCISE 8

The climber pushes against the rock with her back and the rock pushes on her back with a force of equal magnitude. The rock at her back provides a frictional force that helps hold her against the downward force of gravity. Since she is one the verge of slipping the magnitude of the force of friction on her back is given by $f_{\text{back}} = \mu_{\text{back}} N_{\text{back}}$, where N_{back} is the normal force of the rock on her back. A similar statement can be made about the forces at her feet: $f_{\text{shoes}} = \mu_{\text{shoes}} N_{\text{shoes}}$. Solve Newton's second law, with the acceleration equal to zero, for N_{back} and N_{shoes}.

(a) Draw a free-body diagram for the climber. Indicate and label the two normal forces N_{back} and N_{shoes}, the two forces of friction f_{back} and f_{shoes}, and the force of gravity mg. Take the x axis to be horizontal and the y axis to be vertical.

(b) Write Newton's second law in component form:

$$N_{\text{shoes}} - N_{\text{back}} = 0$$

and

$$f_{\text{shoes}} + f_{\text{back}} - mg = 0\,.$$

Substitute $f_{\text{shoes}} = \mu_{\text{shoes}} N_{\text{shoes}}$ and $f_{\text{back}} = \mu_{\text{back}} N_{\text{back}}$, then solve simultaneously for N_{shoes} and N_{back}. According to Newton's third law the magnitude of the push of the feet against the rock is N_{shoes} and the magnitude of the push of the back against the rock is N_{back}.

(c) You want to calculate f_{shoes}/mg. Use $f_{\text{shoes}} = \mu_{\text{shoes}} N_{\text{shoes}}$.

Take the x axis to be parallel to the slide and positive down the slide. Place the origin at the point where the pig starts (from rest). If x is the length of the slide and t is the time the pig takes to slide down it, then $x = \frac{1}{2}at^2$, where a is the acceleration of the pig. Now let a_f be the acceleration when friction is present and let a_{nf} be acceleration for a frictionless slide. Let t be the time of the slide when the slide is frictionless. Then $x = \frac{1}{2}a_{nf}t^2$ and $x = \frac{1}{2}a_f(2t)^2$. This means that $a_{nf} = 4a_f$.

Use Newton's second law to find expressions for a_f and a_{nf}. Draw a free-body diagram for the pig sliding on a slide with friction. Three forces act on the pig: the force of gravity mg, down, the normal force N of the slide, perpendicular to the slide, and the force of kinetic friction $\mu_k N$, parallel to the slide and up the slide. Here m is the mass of the pig and μ_k is the coefficient of kinetic friction between the slide and the pig.

The x component of Newton's second law is

$$mg\sin\theta - \mu_k N = ma_f\,,$$

where θ is the angle the slide makes with the horizontal.

Take the y axis to be in the direction of the normal force. The y component of Newton's second law is

$$N - mg\cos\theta = 0\,.$$

Use this equation to find $N = mg\cos\theta$, then replace N in the x component equation with $mg\cos\theta$. Solve the resulting equation for a_f.

You can find an expression for a_{nf} by setting μ_k equal to zero in the expression you found for a_f. Finally, set a_{nf} equal to $4a_f$ and solve for μ_k.

(a) When the applied force is a minimum, the force of friction is up the plane (helping the applied force) and has its maximum value $\mu_s N$. Use Newton's second law, with the acceleration equal to zero, to solve for F.

Draw a free-body diagram for the block. Indicate and label the force of friction f, the applied force F, the normal force N, and the weight mg. Take the x axis to be up the plane and the y axis to be normal to the plane. Newton's second law yields

$$f + F - mg\sin\theta = 0$$

and

$$N - mg\cos\theta = 0\,.$$

Replace f with $\mu_s N$ in the first equation. Solve the second equation for N and use the resulting expression to substitute for N. Solve for F.

(b) As F is increases the force of friction diminishes to zero, then it increases in the other direction (down the plane) until it reaches its greatest possible magnitude, $\mu_s N$. Then the block starts to slide. Repeat the calculation above, but with the force of friction down the plane.

(c) The Newton's second law equations are exactly like those for part (b) (the frictional force is down the plane) but now $f = \mu_k N$.

Block A is pulled down the incline by the force of gravity and is restrained by the force of friction and the tension force of the rope. The components of these forces parallel to the incline must sum to zero since the velocity of the block is constant. Similarly block B is pulled upward by the tension force of the rope and is restrained by the force of gravity. These forces must also sum to zero. The tension force depends on the mass of block B: the greater the mass the greater the tension force. Thus, the mass of B must be just right to bring about the proper tension force to just balance all other forces on A.

Draw a free-body diagram for block B. Show the force of gravity ($m_B g$, down) and the tension force of the rope (T, up). Take the axis to be positive in the upward direction and write the Newton's second law equation:

$$T - m_B g = 0\,.$$

Draw a free-body diagram for block A. Show the force of gravity ($m_A g$, down), the normal force (N, perpendicular to the plane), the force of friction (f, parallel to the plane and up the plane), and the tension force of the rope (T, parallel to the plane and up the plane). Take the x axis to be parallel to the plane and positive down the plane; take the y axis to be perpendicular to the plane. Write the second law equations:

$$m_A g \sin\theta - T - f = 0$$

and

$$N - m_A g \cos\theta = 0\,,$$

where θ is the angle of the plane above the horizontal.

Replace f with $\mu_k N$ and solve the three equations for m_B. From the first equation $T = m_B g$. Substitute this expression into the other two equations. You should now have

$$N - m_A g \cos\theta = 0$$

and

$$m_A g \sin\theta - m_B g - \mu_k m_A g \cos\theta = 0\,.$$

Substitute $N = m_A g \cos\theta$, from the first equation, into the second equation and solve for m_B. You should get

$$m_B = (\sin\theta - \mu_k \cos\theta) m_A\,.$$

Evaluate this expression.

A large force F clearly holds the smaller block against the larger block. The force of static friction between the blocks balances the force of gravity on the smaller block. If F is reduced, the force of the smaller block on the larger block is reduced, the reaction force of the larger block on the smaller block is reduced, and the maximum allowable force of static friction is reduced. When it becomes less than the force of gravity on the smaller block, that block falls. If the minimum force is applied, the force of friction is given by $f = \mu_s N$, where N is the (horizontal) normal force of the larger block on the smaller block.

Draw a free-body diagram for the smaller block. The forces acting on this block are the force of gravity mg (down), the force of friction f (up), the applied force F (right), and the normal force N exerted by the larger block (left). Take the x axis to be horizontal and the y axis to be vertical. Write the Newton's second law equations:

$$F - N = ma$$

and

$$f - mg = 0\,.$$

Since $f = mg$ and $f = \mu_s N$, you should obtain $N = mg/\mu_s$ and $F - mg/\mu_s = ma$.

Now, consider the two blocks as a single object and write Newton's second law as

$$F = (m + M)a\,.$$

Solve this equation and $F - mg/\mu_s = ma$ simultaneously for F. The other unknown quantity is the acceleration. It must be eliminated.

[ans: 490 N]

(a) Consider the 25-car train to be a single object and draw a free-body diagram for it. Two horizontal forces act on it: the tension force T of the coupling with the locomotive, forward, and the frictional force, backward. Take forward to the positive direction and let N be the number of cars in the train, m be the mass of each car, and a be the acceleration of the train. The horizontal component of Newton's second law is then

$$T - 250Nv = Nma\,.$$

Solve for T. When you substitute numerical values you must convert $30\,\mathrm{km/h}$ to meters per second.

(b) Draw a free-body diagram for the set of 25 cars as a single object on an incline that makes the angle θ with the horizontal. The tension force of the coupling is parallel to the incline and up the incline, the normal force of the incline is perpendicular to the incline, and the force of gravity is downward.

Take the x axis to be parallel to the incline and positive up the incline. The acceleration of the train is zero since the incline is the steepest that the locomotive can negotiate. The x component of Newton's second law is

$$T - Nmg\sin\theta - 250Nv = 0\,.$$

Solve for $\sin\theta$, then θ.

There is a compromise here. As the angle of the cable is increased the forward component of the force it exerts decreases but so does the normal force and the maximum possible force of static friction. This means that a smaller component of the applied force is required to start the box moving. If $f_{\text{s max}}$ decreases more than the forward component of the cable force, then it is advantageous to hold the cable at some angle other than horizontal.

Draw a free-body diagram for the box. Four forces act: the force F applied by the cable, at an angle θ above the horizontal, the force of gravity mg, down, the frictional force f, horizontal and opposite to the direction that box would move if F were great enough, and the normal force N of the floor, up.

Assume the box is not moving, so its acceleration is zero. Also assume it is on the verge of sliding, so $f = \mu_s N$, where μ_s is the coefficient of static friction.

The horizontal component of Newton's second law for the box is

$$F\cos\theta - \mu_s N = 0$$

and the vertical component is

$$N - mg - F\sin\theta = 0\,.$$

Use the second equation to substitute for N in the first. You should get

$$F\cos\theta - \mu_s mg + \mu_s F\sin\theta = 0\,.$$

The mass of sand that can be moved is

$$m = \frac{F\cos\theta + \mu_s F\sin\theta}{\mu_s g}\,.$$

This is a maximum when $f\cos\theta + \mu_s F\sin\theta$ is a maximum. Set the derivative of $F\cos\theta + \mu_s F\sin\theta$ with respect to θ equal to zero, divide through by $\cos\theta$, and replace $\sin\theta/\cos\theta$ with $\tan\theta$. Solve for $\tan\theta$, then for θ.

(b) Substitute the value you found for θ in part (a) and $F = 1100\,\text{N}$ into the expression you found for m. Multiply by g to obtain the weight.

Chapter 6 Hint for Exercise 34

According to Eq. 6–16 the terminal speed is given by

$$v_t = \sqrt{\frac{2mg}{C\rho A}}\,,$$

where m is the mass of the diver, C is the drag coefficient, and A is the effective cross-sectional area. Let v_{tf} be the terminal speed and A_f be the effective cross-sectional area for the faster position and let V_{ts} be the terminal speed and A_s be the effective cross-sectional area for the slower position. Divide the expression for v_{tf} by the expression for v_{ts} to obtain

$$\frac{v_{tf}}{v_{ts}} = \sqrt{\frac{A_s}{A_f}}\,.$$

Solve for A_s/A_f.

Draw a free-body diagram for the car at the top of its circular path. There are two forces on it: the force T of the boom and the force mg of gravity. You do not know the direction of the boom force but you may assume it is upward. The sign of your answer will tell you the direction. Take up to be the positive direction. Then Newton's second law for the car is

$$T - mf = ma\,,$$

where a is the acceleration of the car. Since the car is in uniform circular motion the magnitude of its acceleration is v^2/r, where v is its speed and r is the radius of its circular path. The acceleration vector points from the car toward the center of the path. When the car is at the top this direction is downward, so $a = -v^2/r$. Use this to substitute for a in the second law equation. You should obtain

$$T - mg = -\frac{mv^2}{r}\,.$$

Solve for T. If your answer is positive the force of the boom is upward; if it is negative the force of the boom is downward.

(a) The forces on the bicyclist and bicycle are the downward force of gravity, the upward normal force of the road, and the horizontal frictional force of the road. The acceleration is horizontal and toward the center of the circular path. Since the bicyclist is traveling with constant speed the magnitude of the acceleration is given by $a = v^2/r$, where v is the speed and r is the radius of the path. Newton's second law yields

$$F_h = \frac{mv^2}{r} \,,$$

where m is the mass of the bicyclist and F_h is the horizontal component of the total force on the bicyclist. The only horizontal force is the frictional force of the road.

(b) You must find the normal force of the road. Use the vertical component of Newton's second law. The horizontal component of the total force of the road is the frictional force and the vertical component is the normal force. The magnitude of the total force is the square root of the sum of the squares of its components and the tangent of the angle it makes with the horizontal is the vertical component divided by the horizontal component.

Chapter 6 Hint for Problem 44

When the hand strap makes a nonzero angle with the vertical each part of the strap exerts a force on neighboring parts and that tension force has a horizontal component. It is this component that brings about the centripetal acceleration and causes the strap to go around the curve with the streetcar.

Draw a free-body diagram for a small part of the strap at the bottom end. Two forces act on it: the tension force T, at an angle θ to the vertical, and the force of gravity mg, down. Here m is the mass of the part of the strap that is being considered. The vertical component of Newton's second law is

$$T\cos\theta - mg = 0$$

and the horizontal component is

$$T\sin\theta = ma\,,$$

where a is the acceleration of the strap piece. The strap is in uniform circular motion, so $a = v^2/r$, where v is the speed of the strap (and the streetcar) and r is the radius of the circular path. Use one of the equations to eliminate T from the other, solve for $\tan\theta$, and finally solve for θ. When you substitute numerical values you must convert $10\,\mathrm{km/h}$ to meters per second.

(a) The forces on the passenger are the downward force of gravity, the horizontal force of the seat, and the vertical force of the seat. The acceleration of the passenger is horizontal. Use Newton's second law to determine the vertical component of the total force. You already know the horizontal component.

(b) Since the passenger is going around a circle at constant speed his acceleration is given by $a = v^2/r$, where r is the radius of the circle and v is the speed. Newton's second law yields

$$F_h = \frac{mv^2}{r}\,,$$

where m is the mass of the passenger and F_h is the horizontal component of the total force on the passenger. Solve for v.