Chapter 7 Hint for Exercise 6

The force is constant so $W = \vec{F} \cdot \vec{d}$ can be used to compute the work it does. Recall that $\vec{F} \cdot \vec{d} = F_x d_x + F_y d_y + F_z d_z$.

Chapter 7 Hint for Exercise 8

The force is constant and in the positive x direction, the same direction as the motion of the particle. Thus the work done by the force is given by $W = Fd$, where F is the magnitude of the force and d is the magnitude of the displacement.

You need to find the magnitude of the force. According to Newton's second law $F = ma$, where m is the mass of the body and a is it acceleration. At time $t = 0$ the body is at $x = 0$ so its coordinate is given as a function of time by $x = v_0 t + \frac{1}{2}at^2$, where v_0 is its velocity at $t = 0$. At $t = 1.0\,\mathrm{s}$ the body is at $x = 0.2\,\mathrm{m}$ and at $t = 2.0\,\mathrm{s}$ it is at $x = 0.8\,\mathrm{m}$. Thus $0.2\,\mathrm{m} = (1.0\,\mathrm{s})v_0 + \frac{1}{2}(1.0\,\mathrm{s})^2 a$ and $0.8\,\mathrm{m} = (2.0\,\mathrm{s})v_0 + \frac{1}{2}(2.0\,\mathrm{s})^2 a$. Solve these two equations simultaneously for a. You should get $a = 0.40\,\mathrm{m/s^2}$.

Now use $F = ma$ to calculate F and then $W = Fd$ to calculate W. Here $d = 0.8\,\mathrm{m}$.

Chapter 7 Hint for Problem 10

Differentiate the coordinate with respect to time to find an expression for the velocity. Evaluate the expression for $t = 0$ to find the initial velocity v_i; evaluate it for $t = 4.0\,\mathrm{s}$ to find the final velocity v_f. Then calculate the initial kinetic energy $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy $K_f = \frac{1}{2}mv_f^2$. Use the work-kinetic energy theorem $W = K_f - K_i$ to calculate the total work W.

CHAPTER 7 HINT FOR EXERCISE 15

(a) The tension force of the cord is uniform so the cord pulls on the movable pulley with a force of magnitude $2F$, up. This force is transmitted to the canister. Gravity pulls down on the canister with a force of magnitude mg. Since the canister moves with constant velocity, these forces must have equal magnitudes.

(b) If the canister rises a distance d, the portion of the cord between the movable pulley and the ceiling is shorter by d; the portion of the cord between the two pulleys is also shorter by the same amount.

(c) You actually do work on the rope, pulling on it with a force of 98 N as it moves 4.0 cm.

(d) The work done by the force of gravity is given by $-mgd$, where d is the distance the canister moves.

[ans: (a) 98 N; (b) 4.0 cm; (c) 3.9 J; (d) −3.9 J]

(a) Newton's second law can be used to find the force. Since the block moves with constant velocity the net force is 0.

Draw a free-body diagram for the block. Show the force of the worker (F, up the plane), the force of gravity (mg, down), and the normal force (N, perpendicular to the surface). Choose the x axis to be parallel to the plane and the y axis to be perpendicular to it. Write the second law equations:

$$F - mg\sin\theta = 0$$

and

$$N - mg\cos\theta = 0\,.$$

Solve for F. A little trigonometry shows that $\sin\theta = h/L$, where h is the height of the incline and L is its length.

(b), (c), (d), and (e) Each of the forces is constant so you can calculate the work by multiplying the magnitude of the displacement (1.5 m) by the component of the force along the direction of the displacement. For the force of the worker this component is $-270\,\mathrm{N}$, for the force of gravity it is $mg\sin\theta$ (or mgh/L), for the normal force it is 0, and for the net force it is 0.

For each segment of the lift use kinematics to find the acceleration of the rescuee, then use Newton's second law to find the force of the cable and finally use $W = F_{\text{cable}}d$ to find the work done by the cable force.

(a) The relationship between the final speed v and the distance d the rescuee is lifted is $v^2 = 2ad$, where a is the acceleration. Thus $a = v^2/2d$. Two forces act on the rescuee: the upward force of the cable and the downward force of gravity (mg). Newton's second law gives

$$F_{\text{cable}} - mg = ma\,.$$

Solve for F_{cable}. Then use $W = F_{\text{cable}}d$.

(b) The acceleration of the rescuee is zero, so the net force acting on him is zero. Thus $F_{\text{cable}} = mg$ and $W = mgd$.

(c) Now $v_0^2 = -2ad$, where v_0 is the initial speed of the rescuee ($5.00\,/\text{s}$). Thus $a = -v_0^2/2d$. The acceleration is downward. Again $F_{\text{cable}} - mg = ma$. Solve for F_{cable}, then use $W = F_{\text{cable}}d$.

CHAPTER 7 HINT FOR PROBLEM 22

(a) and (b) Let x be the distance the spring is compressed. Then, the work done by the weight is given by $W_g = mgx$ and the work done by the spring is given by $W_s = -\frac{1}{2}kx^2$.

(c) Use the work-kinetic energy theorem. The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$, where v_i is the speed of the block just before it hits the spring. The final kinetic energy is zero. The net work is the sum of the work done by the spring and the work done by gravity:

$$W_{\text{net}} = W_s + W_g = -1.80\,\text{J} + 0.29\,\text{J} = -1.51\,\text{J}\,.$$

(d) The work-kinetic energy theorem yields

$$mgx - \frac{1}{2}kx^2 = -\frac{1}{2}mv_i^2$$

where $v_i = 7.0\,\text{m/s}$. Solve this quadratic equation for x.

Chapter 7 Hint for Exercise 24

Calculate the area under the curve, taking care about the sign of the work.

From $x = 0$ to $x = 2\,\mathrm{m}$ the force is constant and the work done is the product of the force and the magnitude of the displacement: $(10\,\mathrm{N})(2.0\,\mathrm{m}) = 20\,\mathrm{J}$). The area under the curve from $x = 2.0\,\mathrm{m}$ to $x = 4.0\,\mathrm{m}$ can be computed if you know that the area of a triangle is half the product of the base and altitude: $\frac{1}{2}(10\,\mathrm{N})(2.0\,\mathrm{m}) = 10\,\mathrm{J}$. From $x = 4.0\,\mathrm{m}$ to $x = 6.0\,\mathrm{m}$ the force is 0 and does no work. From $x = 6.0\,\mathrm{m}$ to $x = 8.0$ the force is opposite to the displacement and the work is negative. Use the formula for the area of a triangle. Add the results for the various segments.

Use the work-kinetic energy theorem: $K_f = K_i + W$, where K_i is the kinetic energy of the body at the beginning of an interval, K_f is the its kinetic energy at the end of the interval, and W is the net work done on the body during the interval.

(a) Here the interval runs from the time the body is at $x = 0$ to the time the body is at $x = 3.0\,\mathrm{m}$. The graph tells us that the work done by the only force acting on the body is zero from the time the body is at $x = 0$ to the time it is at $x = 2.0\,\mathrm{m}$. This is because the force does positive work during the first half of the interval and does an equal amount of negative work during the second half. All you need to consider is the interval from the time the body is at $x = 2.0\,\mathrm{m}$ to the time it is at $x = 3.0\,\mathrm{m}$. The force is constant during this interval, so this work is

$$W = Fd = (-4.0\,\mathrm{N})(1.0\,\mathrm{m}) = -4.0\,\mathrm{J}\,.$$

The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$, where m is the mass of the body and v_i is its speed when it is at $x = 0$.

(b) First calculate the work done by the force as the body moves from $x = 0$ to the point where its kinetic energy is $K_f = 8.0\,\mathrm{J}$. Use $K_f = K_i + W$, where $K_i = \frac{1}{2}mv_i^2$ and $v_i = 4.0\,\mathrm{m/s}$. The final point will be beyond $x = 2.0\,\mathrm{m}$, so you can use $W = Fd$, where $F = -4.0\,\mathrm{N}$ and d is the distance traveled beyond $x = 2.0\,\mathrm{m}$. That is, you will need to add $2.0\,\mathrm{m}$ to the result you obtain for d.

(c) The force does positive work as the body moves from $x = 0$ to $x = 1.0\,\mathrm{m}$. Thereafter it does negative work. The body has maximum kinetic energy when it is at $x = 1.0\,\mathrm{m}$. The work done by the force is the area under the curve. Since the two axes and the line representing the force form a triangle the area is half the product of the base ($1.0\,\mathrm{m}$) and the altitude ($4.0\,\mathrm{N}$). Add this work to the initial kinetic energy to find the final kinetic energy.

(a) Use the work-kinetic energy theorem: $K_f = K_i + W$, where K_i is the kinetic energy of the block at the beginning of an interval (when it is at $x = 0$), K_f is its kinetic energy at the end of the interval (when it is at $x = 2.0\,\mathrm{m}$), and W is the net work done on the block during the interval. Since the block starts from rest and since the given force is the only force acting on the block with a component in the direction of motion, the kinetic energy of the block when it passes $x = 2.0\,\mathrm{m}$ is equal to the work done by the force. The force is variable; it depends on the coordinate x of the block, so you must use an integral to calculate the work:

$$W = \int_0^{2.0\,\mathrm{m}} F_x \,\mathrm{d}x = \int_0^{2.0\,\mathrm{m}} (2.5 - x^2)\,\mathrm{d}x\,.$$

(b) The force does positive work for a while, then does negative work. The kinetic energy of the block increases at first, then decreases. Its maximum kinetic energy occurs at the point where the force starts doing negative work. That is, it occurs at the point where the force switches from positive to negative. The value of the force is zero there. Solve $2.5 - x^2 = 0$ for x, then evaluate the integral for the work, with the value you found for x as the upper limit. Since the block starts from rest the work equals its final kinetic energy.

Chapter 7 Hint for Exercise 32

(a) Evaluate the scalar product $P = \mathbf{F} \cdot \mathbf{v} = F_x v_x + F_y v_y + F_z v_z$.

(b) Let $\mathbf{v} = v_y\,\mathbf{j}$ and set $\mathbf{F} \cdot \mathbf{v} = -12\,\mathrm{W}$. Solve for v_y.

Chapter 7 Hint for Problem 34

(a) The speed of the rope (and skier) is constant for either rope speed. The work-kinetic energy theorem tells us that the net work done on the skier is zero. Two forces do work: force of the tow rope and the force of gravity. The two works must have the same magnitudes and opposite sign. The work done by gravity is mgh, where m is the mass of the skier and h is the vertical distance the skier rises. Since the distance the skier moves is the same for the two speeds, the distance the skier rises is the same, the work done by gravity is the same, and the work done by the tow rope is the same.

(b) and (c) The rate with which the tow rope does work is given by $P = Fv$, where F is the force of the tow rope and v is the speed of the skier. Use $W = Fd$, where d is the distance traveled by the skier along the slope, to find the force of the tow rope.

(a) The rate with which the force of the spring does work is given by $P = F_s v$, where F is the force of the spring on the bolt and the v is the velocity of the bolt. When the bolt is at the equilibrium position the force of the spring is zero.

(b) Since the force of the spring is given by $F = -kx$, where k is the spring constant and x is the displacement of the bolt from the equilibrium position, the rate with which the spring does work is given by $P = F_s v = -kxv$. Here $x = -0.10\,\text{m}$. You need to find the velocity of the bolt.

When the bolt is at the equilibrium position its kinetic energy is $K_i = 10\,\text{J}$. As the bolt moves from the equilibrium position to $x = -0.10\,\text{m}$ the force of the spring does work $W = -\frac{1}{2}kx^2$. This is the only work being done on the bolt. According to the work-kinetic energy theorem the kinetic energy of the bolt when it is at $x = 0.10\,\text{m}$ is given by

$$K_f = K_i + W = K_i - \frac{1}{2}kx^2\,.$$

Replace K_f with $\frac{1}{2}mv^2$, where v is the velocity of the bolt when it is at $x = 0.10\,\text{m}$. Solve for v. Since the bolt is moving away from the equilibrium position, v is negative.

Chapter 7 Hint for Problem 40

(a) Use the work-kinetic energy theorem. You can use the given data to calculate the initial and final kinetic energies.

(b) and (c) Use $P = Fv$ to compute the rate with which work is done and $F = ma$ to compute the force. Here

$$a = \frac{v}{t} = \frac{10\,\mathrm{m/s}}{3.0\,\mathrm{s}} = 3.33\,\mathrm{m/s^2}\,.$$

At the end of the interval $v = 10\,\mathrm{m/s}$ while at the midpoint of the interval $v = at = 3.33 \times 1.5 = 5.0\,\mathrm{m/s}$.