## Chapter 9 Hint for Problem 6

Symmetry indicates that the center of mass of the slab lies on the plane halfway through the thickness of the slab and halfway from front to back. You need to calculate its position along an axis that runs from left to right through the center of the sample. Place an $x$ axis $1.40\,\mathrm{cm}$ above the lower face and $6.5\,\mathrm{cm}$ from the front edge, with its origin at the left edge of the slab. Replace the iron with a particle of mass $M_i = \rho_i V_i$, located at its center: on the $x$ axis at $x = 5.5\,\mathrm{cm}$. Here $\rho_i$ is the density of iron and $V_i$ is the volume of iron in the slab. Replace the aluminum with a particle of mass $M_a = \rho_a V_a$, located at its center: on the $x$ axis at $x = 16.5\,\mathrm{cm}$. Here $\rho_a$ is the density of aluminum and $V_a$ is the volume of aluminum in the slab. Find the center of mass of the two replacement particles.

# Chapter 9 Hint for Problem 7

First consider the hydrogen atoms alone. Their center of mass is at the center of the triangle. To see this, replace two of the atoms with an atom of twice the mass at the midpoint of a triangle side (at A). Now, draw the line from the opposite vertex to A. Since there is twice as much mass at A as at the vertex, the center of mass of the three atoms lies on the dotted line, one third the distance to the vertex (that is, at B). This is directly beneath the nitrogen atom.

Replace the three atoms with one of three times the mass, at B. Find the coordinates of the center of mass of this atom and the nitrogen atom. Place the origin of a coordinate system at the nitrogen atom and take the $z$ axis to be positive downward. If $d$ is the distance from the nitrogen to the plane of hydrogens below, the $z$ coordinate of the center of mass is given by $3m_H d/(3m_H + m_N)$. The distance $d$ can be found by applying the Pythagorean theorem to the triangle formed by the nitrogen, the center of the base triangle and a hydrogen atom.

[ans: $6.75 \times 10^{-12}$ m directly below the nitrogen atom]

## Chapter 9 Hint for Problem 8

Replace each of the five sides with a single particle with mass equal to the mass of the side and positioned at the center of mass of the side. Let $m$ be the mass of a side. The center of mass of the side is at the geometrical center of the side. The center of mass of the box is at the same position as the center of mass of the five particles. Use

$$x_{\text{com}} = \frac{1}{M} \sum m x_i \,,$$

$$y_{\text{com}} = \frac{1}{M} \sum m y_i \,,$$

and

$$z_{\text{com}} = \frac{1}{M} \sum m z_i$$

to compute the coordinates of the center of mass of the box. Here $M$ $(= 5m)$ is the mass of the box and $x_i$, $y_i$, and $z_i$ are the coordinates of particle $i$.

# Chapter 9 Hint for Exercise 10

The net external force acting on the system consisting of the two skaters and the pole is zero, so the velocity of the center of mass does not change. The forces of the skaters on the pole and of the pole on the skaters are all internal forces and sum to zero. Since the center of mass was at rest to begin with, it remains at the same position. The skaters meet at their center of mass. To find how far the 40 kg skater moves, calculate the original distance from that skater to the center of mass.

Place the origin of the coordinate system at the 40 kg skater. The 65 kg skater is then at $x = 10\,\mathrm{m}$. Calculate the coordinate of the center of mass.

(a) Use kinematics to find the coordinates of the stones, then calculate the coordinate of the center of mass.

Take the $y$ axis to be downward with the origin at the release point. The coordinate of the first stone is then given by

$$y_1 = \frac{1}{2}gt^2$$

and the coordinate of the second stone is given by

$$y_2 = \frac{1}{2}g(t - t_0)^2 \,,$$

where $t_0 = 200\,\mathrm{ms}$. Evaluate these expressions for $t = 300\,\mathrm{ms}$, then use

$$y_{\mathrm{com}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \,,$$

with $m_2 = 2m_1$.

(b) The velocity of the first stone is given by $v_1 = gt$ and that of the second is given by $v_2 = g(t - t_0)$. Evaluate these expressions for $t = 300\,\mathrm{ms}$, then use

$$v_{\mathrm{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \,.$$

[ans: (a) 28 cm; (b) 2.3 m/s]

The net external force on the system consisting of the canoe and its two passengers is zero, so you know that the velocity of the center of mass is constant. Furthermore, the canoe and its passengers are not moving initially, so the velocity of the center of mass is zero. The center of mass remains at the same place relative to the log when the passengers change places.

Place a coordinate axis along the length of the canoe and replace the canoe and each of its passengers with a particle. The particle that replaces the canoe has a mass that is equal to the mass of canoe and is positioned at the center of the canoe. The particle that replaces either of the passengers has a mass that is equal to the mass of the passenger and is located at the position of the passenger.

Write an expression for the coordinate of the center of mass of the system when the canoe and passengers are at their initial positions. If the center of the canoe is at $x = x_{c0}$, then Ricardo is at $x = x_{c0} + \frac{1}{2}\ell$ and Carmelita is at $x = x_{c0} - \frac{1}{2}\ell$, where $\ell$ is the distance between the seats. Write another expression for the center of mass of the system after the passengers change places. If the center of the canoe is at $x = x_{c1}$, then Ricardo is at $x = x_{c1} - \frac{1}{2}\ell$ and Carmelita is at $x = x_{c1} + \frac{1}{2}\ell$.

Equate the two expressions for the coordinate of the center of mass and solve for Carmelita's mass.

## Chapter 9 Hint for Problem 24

(a) Let $\vec{v}_b = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$ be the velocity of the ball before it bounces. Then, $\vec{v}_a = v_x\,\hat{\imath} - v_y\,\hat{\jmath}$ is its velocity afterwards. From the diagram both $30^\circ$ and $\theta$ have the same tangent, namely $v_x/v_y$. Furthermore, they are in the same quadrant.

(b) Calculate the initial linear momentum:

$$\vec{p}_b = mv_x\,\hat{\imath} + mv_y\,\hat{\jmath} = (mv\sin\theta)\,\hat{\imath} + (mv\cos\theta)\,\hat{\jmath}\,.$$

Calculate the final linear momentum:

$$\vec{p}_a = (mv\sin\theta)\,\hat{\imath} - (mv\cos\theta)\,\hat{\jmath}\,.$$

Finally, calculate the difference $\vec{p}_a - \vec{p}_b$.

# CHAPTER 9 HINT FOR PROBLEM 25

For parts (a) and (b) use

$$\vec{p} = m\vec{v} = m\,\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

and for part (c) use

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\,.$$

The direction of the linear momentum is the direction of motion.

[ ans: (a) $(-4.0 \times 10^4\,\mathrm{kg \cdot m/s})\,\hat{\imath}$; (b) west; (c) 0 ]

## Chapter 9 Hint for Exercise 28

Since the velocity of the center of mass does not change, the linear momentum of the system consisting of the two blocks is conserved. Since the center of mass is at rest, the linear momentum of the system is zero. Solve $m_1v_1 + m_2v_2 = 0$ for $v_2$.

## CHAPTER 9 HINT FOR EXERCISE 32

The net external force on the system consisting of the flatcar and the man is zero, so the total linear momentum of the system is conserved. Take the positive direction to be to the right. The horizontal component of the total linear momentum before the man starts running is $(m_m + m_f)v_0$, where $m_m$ is the mass of the man, $m_f$ is the mass of the flatcar, and $v_0$ is the velocity of the flatcar (and man). After the man starts running, the total linear momentum is $m_m v_m + m_f v_f$, where $v_m$ is the velocity of the man and $v_f$ is the velocity of the flatcar. These velocities are measured relative to the ground.

The velocities of the man and flatcar are related by $v_m = v_f - v_{\rm rel}$, where $v_{\rm rel}$ is the speed of the man relative to the flatcar. Thus the total linear momentum after the man starts running is $m_m v_f - m_m v_{\rm rel} + m_f v_f$.

Equate the two expressions for the total linear momentum and solve for $v_f - v_0$. Use $W = m_f g$ and $w = m_m g$ to write the result in terms of the weights.

Take the system to consist of the two particles and the spring. Assume that no forces act on the particles except the forces of the spring. Then the total linear momentum of the system is conserved and since the particles are initially at rest the total linear momentum is zero. Thus $m_A v_A + m_B v_B = 0$, where $m_A$ is the mass of particle A, $m_B$ is the mass of particle B, $v_A$ is the velocity of particle A after it is free of the spring, and $v_B$ is the velocity of particle B after it is free of the spring. Use conservation of linear momentum equation and $m_A = 2.00 m_B$ to show that $v_B = -2v_A$.

Since the energy originally stored in the spring is converted to the kinetic energy of the particle, so

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = 60\,\mathrm{J}\,.$$

Substitute $m_B = m_A/2$ and $v_B = -2v_A$, then solve for $\frac{1}{2}m_A v_A^2$. This is the kinetic energy of particle A after it is free of the spring. The kinetic energy of particle B is 60 J minus the kinetic energy of particle A.

# CHAPTER 9 HINT FOR PROBLEM 40

Since no external forces act, the total linear momentum is conserved. It has the same value after the explosion as before. Let $m$ be the original mass and let $v$ be its velocity. Take the masses of the chunks to be $m_1$ and $m_2$ with velocities $v_1$ and $v_2$ after the explosion. Then

$$mv = m_1 v_1 + m_2 v_2\,.$$

If $Q$ is the energy added in the explosion, then

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}mv^2 + Q\,.$$

Solve these equations simultaneously for $v_1$ and $v_2$.

The algebra is simplified considerably if you first make the substitutions $m_1 = m/2$ and $m_2 = m/2$. Use the conservation of linear momentum equation to substitute for one of the unknown velocities in the energy equation. You will obtain a quadratic equation for the other velocity. Either solution is valid.

# CHAPTER 9 HINT FOR EXERCISE 42

(a) No external forces act on the rocket-fuel system so $T = Ma$, where $T$ is the thrust, $M$ is the mass, and $a$ is the acceleration. Solve for $T$.

(b) The thrust is given by

$$T = Rv_{\text{rel}} \,,$$

where $R$ is the rate of fuel exhaust and $v_{\text{rel}}$ is its speed relative to the rocket. Solve for $R$.

## CHAPTER 9 HINT FOR EXERCISE 46

Suppose that at some instant the mass of the car and its load is $M$. In the short time interval $\Delta t$ suppose grain of mass $\Delta M$ is added to the car. The horizontal component of the grain's velocity is initially 0 but during loading becomes $v$, the velocity of the car. The linear momentum of the car-grain system initially is $Mv$ and, after time $\Delta t$, is $(M+\Delta M)v$. The rate of change of linear momentum is

$$\frac{\Delta P}{\Delta t} = v\frac{\Delta M}{\Delta t}$$

and in the limit as $\Delta t$ becomes small this equation becomes

$$\frac{\mathrm{d}P}{\mathrm{d}t} = v\frac{\mathrm{d}M}{\mathrm{d}t}\,.$$

The external force that must be applied is

$$F = \frac{\mathrm{d}P}{\mathrm{d}t} = v\frac{\mathrm{d}M}{\mathrm{d}t}\,.$$

# Chapter 9 Hint for Problem 54

If $f$ is the frictional force and $d$ is the stopping distance, then the change in the kinetic energy of the center of mass is given by

$$K_{\text{com }f} - K_{\text{com }i} = -fd\,.$$

The initial kinetic energy of the car and passengers is $K_{\text{com }i} = \frac{1}{2}mv_i^2$, where $v_i$ is the initial speed of the car. The final kinetic energy is $K_{\text{com }f} = 0$. Solve for $d$.