You may skip one and only one question or problem.

In order to take advantage for this you must write the number of the question or problem here and state if it is a question (Q) or Problem (P).

Number of problem or question not to be graded

Points for each question and problem are indicated in red with the amount being spread equally among parts (a,b,c etc). Be sure to show all your work. Use the back of the pages if necessary.
Question 1 (10 points). What is the direction of the force for each situation below. Draw the force and describe it too (up, down, left, right, into page, out of page). If there is no force state that there is no force.

A

proton

Ans: The direction of the force is left.

B

electron

Ans: The direction of the force is left. (A and B can be confirmed by right-hand rule.)

C

Ca^{2+} ion

Ans: There is no force, because the velocity direction of ion is parallel with the direction of magnetic field.
Question 2 (10 points). For each of the following currents drawn below (in blue), state which way the magnetic field points at the point P indicated (in red). Assume current wires are infinitely long. Draw the field and describe it too (up, down, left, right, into page, out of page).

A.

\[ \text{Ans: The magnetic field at the point P is into page.} \]

B.

\[ \text{Ans: The magnetic field at the point P is into page.} \]

C.

\[ \text{Ans: The magnetic field at the point P is downward.} \]
Question 3 (10 points). A square, flat loop of wire is pulled at constant velocity through a region of uniform magnetic field directed perpendicular to the plane of the loop as shown in the figure below.

(a) Does a current flow in the loop? If so, is it clockwise or counter clockwise. [Note that the loop does not leave the magnetic field].

Ans: No. The magnetic flux through the coil is constant in time, so the induced emf is zero.

(b) Is there a charge separation that occurs in the loop? If so, where does positive charges accumulate – top, bottom, right, left?

Ans: Yes, the positive charges accumulate at top. (The positive test charges in the leading and trailing sides of the square experience a force that is in direction toward the top of the square. The charges migrate upward to give positive charge on the top of the square until there is a downward electric field large enough to prevent more charge displacement.)

Now consider the bar shown in the figure below that moves on rails to the right with a velocity $\vec{v}$, in a uniform, constant magnetic field that is directed out of the page.

(c) Is there an induced current? If so is it clockwise or counterclockwise?

Ans: Yes, the induced current is clockwise. (By the magnetic force law $\vec{F}=q(\vec{v}\times\vec{B})$: the positive charges in the moving bar will feel a magnetic force in direction downward toward the bottom end of the bar. These charges will move downward and therefore clockwise in the circuit.)

(Alternatively, Lenz’s Law can be applied to explain: When the bar moves to the right, the area of the loop increases as well as the magnetic flux through it. By Lenz’s Law, a current will be induced in the loop in such a direction that the magnetic field produced by this current is opposite the change of magnetic flux. In this case, the induced current is clockwise to produce an inward secondary magnetic field.)

(d) Is there a force required to keep the bar moving?
Ans: Yes. The current induced in the bar experiences a force to the left in the magnetic field that tends to slow the bar, therefore, an external force is required to keep the bar moving at constant speed to the right.

Now suppose a bar magnet is held above the center of a wire loop lying in the horizontal plane, as shown in the figure below. The south end of the magnet is toward the loop.

(e) Is there a current induced in the loop? If so, is it clockwise or counter clockwise as seen from above?

Ans: Yes, the induced current is clockwise as the magnet moves in. (As the bar magnet approaches the loop from above, with its south end downward, the magnetic flux through the area enclosed by the loop is directed upward and increasing in magnitude. To oppose this increasing upward flux, the induced current in the loop will flow clockwise, as seen from above.

Later on, after the bar magnet has passed through the plane of the loop, and is departing with its north end upward, a decreasing flux is directed upward through the loop. To oppose this decreasing upward flux, the induced current in the loop flows counterclockwise as seen from above, producing flux directed upward through the area enclosed by the loop.)
Problem 1 (15 points). A proton, \( q = 1.6 \times 10^{-19} \text{ C} \), moves perpendicular to a uniform magnetic field \( \mathbf{B} \) at a speed of \( 1.00 \times 10^7 \text{ m/s} \) and experiences an acceleration of \( 2.00 \times 10^{13} \text{ m/s}^2 \) in the positive \( x \) direction when its velocity is in the positive \( z \) direction.

(a) Determine the magnitude and direction of the field.

(b) What if an electron were to now move into this same field travelling in the \( x-y \) plane with velocity of \( 1.0 \times 10^7 \text{ m/s} \) and making an angle of \( \theta = 30 \) degrees below the \( x \)-axis (as shown). What would the magnitude and direction of the force on the electron due to the magnetic field?

Drawing for part b

\[
\begin{align*}
\text{Ans:} \\
(a) \quad & F = ma = (1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{13} \text{ m/s}^2) = 3.34 \times 10^{-14} \text{ N} = qvB \sin 90^\circ \\
& B = \frac{F}{qv} = \frac{3.34 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^7 \text{ m/s})} = 2.09 \times 10^{-2} \text{ T} = 20.9 \times 10^{-3} \text{ T} = 20.9 \text{ mT} \\
& \text{The right-hand rule shows that } B \text{ must be in the } -y\text{-direction to yield a force in the } +x\text{-direction when } v \text{ is in the } z\text{-direction. Therefore } B = -20.9j \text{ mT.} \\

(b) \quad & F = q(v \times B) = (-1.60 \times 10^{-19} \text{ C}) \begin{vmatrix} i & j & k \\ \frac{\sqrt{3}}{2}v & -\frac{1}{2}v & 0 \\ 0 & -B & 0 \end{vmatrix} = (-1.60 \times 10^{-19} \text{ C}) \frac{\sqrt{3}}{2} (1.0 \times 10^7 \text{ m/s})(-2.09 \times 10^{-2} \text{ T}) = 2.90 \times 10^{-14} \text{ k N} \\
& \text{The direction of the force is on the } +z\text{-direction.} \\
& \text{(By right-hand rule, magnetic force is along } +z \text{ direction. The magnitude is } qvB \sin 60^\circ).} 
\end{align*}
\]
Problem 2 (15 points). A current path shaped as shown in the figure produces a magnetic field at $P$, the center of the arc. The arc subtends an angle of $\theta = 30.0^\circ$ and the radius of the arc is 0.600 m, and the current is 3.00 A.

(a) What is the magnitude and direction of the magnetic field at point $P$ due to the top, straight segment of current (labeled 1.), if that segment does indeed produce a field at $P$. If the field due to the segment is zero state so.

(b) What is the magnitude and direction of the magnetic field at point $P$ due to the bottom, straight segment of current (labeled 2.), if that segment does indeed produce a field at $P$. If the field due to the segment is zero state so.

(c) What is the magnitude and direction of the magnetic field at point $P$ due to the arc of current (labeled 3.), if that segment does indeed produce a field at $P$. If the field due to the segment is zero state so.

Ans: (a) The total magnetic field at $P$ by current 1 is zero. Use Biot-Savart law: the magnetic field at $r$ by a segment of current, $dB = \frac{\mu_0 Ids \times r}{4\pi r^3}$. For the top, straight current 1, every segment along it has a direction parallel to that pointed. To point $P$, such that $ds \times r = 0$ and $dB = 0$.

(b) The total magnetic field at $P$ by current 2 is zero. The reason is the same as (a).

(c) $B = \int_{arc} dB = \frac{\mu_0 I}{4\pi r^2} \theta$, where $|dB| = \frac{\mu_0 Ids}{4\pi r^2}$. For the arc, $ds$ is along the tangent direction at any point and $r$ is along the radial direction.

More directly $B = \frac{\mu_0 I}{4\pi r^2} \int ds \times r = \frac{\mu_0 I}{4\pi r^2} s$

$s = r\theta = (0.6 \text{ m})(30^\circ)(2\pi/360^\circ) = 0.314 \text{ m}$

$B = (10^{-7})((3)(0.314)/(0.6)^2) = 262 \text{ nT into page by rh rule.}$
Problem 3 (15 points). The figure below is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is $I_1 = 1.00$ A out of the monitor, and the current in the outer conductor is $I_2 = 3.00$ A into the monitor. (a) Determine the magnitude and direction of the magnetic field at point a (b) Determine the magnitude and direction of the magnetic field at point b.

Ans: (a) At point a, the magnitude $B_1 = \frac{\mu_0 I_1}{2 \pi r_1}$ and the direction is upward. (Apply Ampere’s Law for the magnitude field produced by a current $\oint B \cdot ds = \mu_0 l$. Imagine a concentric circle, $B$ is along the tangent direction at every point, having its center at the axis and radius 1mm. By symmetry, every point at this circle has the same magnitude of magnetic field. 

$$\oint B \cdot ds = B_1 2 \pi r_1 = \mu_0 l_1,$$ 

so $B_1 = \frac{\mu_0 l_1}{2 \pi r_1} = \frac{(2 \times 10^{-7})(1)}{1 \times 10^{-3}} = 2 \times 10^{-4}$ T. By right-hand rule, $B$ at point a is upward.

(b) $\oint B \cdot ds = B_2 2 \pi r_2 = \mu_0 |l_1 - l_2|$, therefore $B_2 = \frac{\mu_0 (l_2 - l_1)}{2 \pi r_2}$, and $B$ at point b is downward.

$$B_2 = \frac{(4 \pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A})(2.00 \text{ A})}{2 \pi (3.00 \times 10^{-3} \text{ m})} = 133 \text{ µT}$$ towards the bottom of the page.
Problem 4 (15 points). A magnetic field directed into the page changes with time according to 
\[ B = 0.03t^2 + 1.40, \]
where \( B \) is in teslas and \( t \) is in seconds. The field has a circular cross section of 
radius \( R = 2.50 \text{ cm} \) (see figure below).

(a) When \( t = 4.00 \text{ s} \) and \( r_2 = 0.020 \text{ m} \), what is the magnitude of the electric field at point \( P_2 \)?

(b) When \( t = 4.00 \text{ s} \) and \( r_2 = 0.020 \text{ m} \), what is the direction of the electric field at point \( P_2 \)?

(c) If there were a circular wire or radius \( R \), centered around the field with a resistance of 1 ohm, what would the current in the wire be and would it be clockwise or counter clockwise.

(d) When \( t = 4.00 \text{ s} \), what is the magnitude and direction of the electric field at point \( P_1 \) with \( r_1 = 5.00 \text{ cm} \)?

Ans: (a) By Faraday’s Law of induction, \( \oint E \cdot ds = \frac{d\Phi_B}{dt} \), where \( \Phi_B = BA = B_{in}(\pi r_2^2) \). By symmetry, \( E \) has the same magnitude at every point on the circle, such that 
\[ \oint E \cdot ds = E \cdot 2\pi r_2 = \pi r_2^2 \frac{d\Phi_{in}}{dt} = \pi r_2^2 (0.06t), \]
therefore,
\[ E = \frac{r_2}{2} (0.06t) = \frac{0.02m}{2} (0.06 \times 4.00s) = 0.0024 \text{ N/C} \]
(b) By Lenz’s Law, the increasing magnetic field will induce an electric field with the trend to 
impede its increase. So \( E \) is tangent to the circle at \( P_2 \) and along counter clockwise direction.

(c) \( \varepsilon = -\frac{d\Phi_B}{dt} \), so \( I = \frac{\pi R^2 (\frac{d\Phi}{dt})}{10\text{ohm}} = \pi R^2 (0.06t) \). The current will increase with the time increasing and the direction is counter clockwise. At 4 seconds this would be 0.47mA.

(d) \( \oint E \cdot ds = E \cdot 2\pi r_1 = \pi R^2 B_{in} \), 
so \( E = \frac{R^2}{2r_1} (0.06t) = \frac{0.025m+0.025m}{2+0.05m} (0.06 \times 4.00s) = 0.0015 \text{ N/C} \)
And \( E \) is tangent to the circle at \( P_1 \) and along counter clockwise direction.
\[ F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2} \]

\[ e = 1.6 \times 10^{-19} \text{ C} \]

\[ E = \frac{|q|}{4\pi\varepsilon_0 r^2} \]

\[ \Delta x = x_2 - x_1, \Delta t = t_2 - t_1 \]

\[ s = \text{(total distance)} / \Delta t \]

\[ \ddot{a} = \Delta v / \Delta t \]

\[ v = v_0 + at \]

\[ x-x_0 = v_0t + \left(\frac{1}{2}\right)at^2 \]

\[ v^2 = v_0^2 + 2a(x-x_0) \]

\[ x-x_0 = \frac{1}{2}(v_0 + v)t \]

\[ x-x_0 = vt - 1/2at^2 \]

\[ \ddot{a} = \frac{d\ddot{v}}{dt} \]

\[ \Delta U = U_f - U_i = -W \]

\[ \Delta V = V_f - V_i = -W/q_0 = \Delta U/q_0 \]

\[ V_f - V_i = -\int_i^f \vec{E}.d\vec{s} \]

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \]

\[ E_s = \frac{\partial V}{\partial s} \]

\[ E = \frac{\Delta V}{\Delta s} \]

\[ Q = CV \]

\[ C = 2\pi\varepsilon_0 \frac{l}{\ln(b/a)} \]

\[ C = 4\pi\varepsilon_0 R \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ (C}^2 / \text{N m}^{-2}) \]

\[ \vec{E} = \vec{F}/q_0 \]

\[ \varepsilon_0\Phi = \varepsilon_0 \int \vec{E}.d\vec{A} = q_{\text{enc}} \]

\[ \vec{v} = \Delta x / \Delta t \]

\[ v = dx/dt \]

\[ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \]

\[ g = 9.8 \text{ m/s}^2 \]

\[ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \]

\[ \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \]

\[ \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \]

\[ \ddot{v} = \Delta\vec{r} / \Delta t, \dot{v} = d\vec{r} / dt \]

\[ \ddot{a} = \Delta\ddot{v} / \Delta t \]

\[ \Delta U = U_f - U_i = -W/q_0 \]

\[ V = -W/q_0 \]

\[ E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z} \]

\[ U = -W = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r_{12}} \]

\[ C = \frac{\varepsilon_0A}{d} \]

\[ C = 4\pi\varepsilon_0 \frac{ab}{b-a} \]

\[ C_{eq} = \sum C_j \text{ (parallel)} \]
\[
\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_j} \quad \text{(series)}
\]

\[
u = \frac{1}{2} \varepsilon_0 E^2
\]

\[I = \frac{dQ}{dt}\]

\[\rho = \frac{1}{\sigma}\]

\[R = \frac{B L}{A}\]

\[P = IV\]

\[P_{\text{emf}} = I\varepsilon\]

\[
\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_j} \quad \text{(parallel)}
\]

\[I = \frac{(\varepsilon/R)e^{\varepsilon t}}{R + r}\]

\[I = \frac{(Q/R)}{e^{RC}}, \quad I_0 = (Q/RC)\]

\[\vec{F} = q\vec{v} \times \vec{B}\]

\[r = mv/qB, \quad \omega = qB/m\]

\[d\vec{F} = I d\vec{s} \times \vec{B}\]

\[\vec{\mu} = N\vec{A}\]

\[B = \mu_0 I/2 \pi r\]

\[F/l = (\mu_0 I_1 I_2)/2\pi a\]

\[\varepsilon = \int \vec{E}.d\vec{s} = -N \frac{d\Phi_B}{dt}\]

\[\varepsilon = Blv\]

\[U = \frac{Q^2}{2C} = \frac{1}{2} CV^2\]

\[C = \kappa C_0\]

\[V = IR\]

\[P = I^2 R = V^2/R\]

\[I = \frac{\varepsilon}{(R + r)}\]

\[R_{\text{eq}} = \sum R_j \quad \text{(series)}\]

\[q(t) = Q(1 - e^{-t/RC})\]

\[q(t) = Qe^{t/RC}\]

\[\vec{F} = I\vec{L} \times \vec{B}\]

\[\tau = \vec{\mu} \times \vec{B}\]

\[U = -\vec{\mu} \cdot \vec{B}\]

\[d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \vec{r}}{r^3}, \quad \mu_0 = 4\pi \times 10^{-7} \text{Tm/A}\]

\[B = \mu_0 nI \quad \text{(solenoid)}\]

\[\int \vec{B}.d\vec{s} = \mu_0 I_{\text{enc}}\]

\[\Phi_B = \int \vec{B}.d\vec{A}\]

\[\Phi_B = \oint \vec{B}.d\vec{A} = 0\]