

Physics 114

Exam 1 Spring 2023

Name: _____

For grading purposes (do not write here):

Question

Problem

1.

1.

2.

2.

3.

3.

Include a statement and signature that you have followed the honor code and not cheated on this exam.

Answer each of the following questions and each of the problems. Points for each question and problem are indicated in red with the amount being spread equally among parts (a,b,c etc). Be sure to show all your work.
Use the back of the pages if necessary.

Question 1. (10 points) Consider two charges $Q_1 = 5\text{C}$ and $Q_2 = -2\text{ nC}$ that are 1 m apart.

- (a) What is the direction of the force by Q_1 on Q_2
- (b) What is the direction of the force by Q_2 on Q_1
- (c) How do the magnitude of these forces compare with each other? Are they the same or different? If one is bigger than the other, which one and by how much.



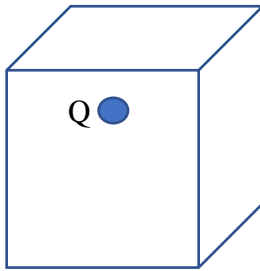
Solutions

- (a) The force is attractive. Q_1 is pulling Q_2 towards itself. **The force is horizontal pointing towards the left.**
- (b) Here, Q_2 is pulling on q_1 and the force is **horizontal to the right.**
- (c) These are action/reaction pairs the forces are **equal in magnitude** and opposite in direction.

Question 2. (10 points) Consider a charge $Q = +2 \mu\text{C}$ that inside the box placed closer to the top of the box than the bottom as shown. The sides of the cube are 2 cm.

- (a) What is the total flux through the cube?
- (b) Can we use Gauss' law to determine the electric field at any of the 6 surfaces of the cube? If not, why not?
- (c) How does the electric flux through the top side of the cube compare to that through the bottom? Are they the same or is one bigger and state which is bigger if that is the case.
- (d) If the sides of the cube are doubled to 4 cm so that each face of the cube becomes further away from the charge, what happens to the total flux through the cube? What happens to the electric field at any point on the surface of the cube compared to before the size of the cube was changed.

Solution. (a) By Gauss' law, the flux is equal the charge inside divided by ϵ_0 . Thus it is $2 \times 10^{-6} / 8.85 \times 10^{-12} = 2.26 \times 10^5 \text{ N}\cdot\text{m}^2/\text{C}$.



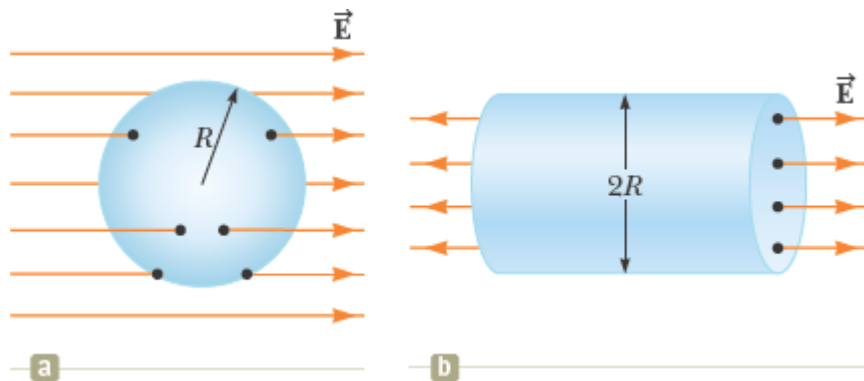
(b) **No**, there is not enough symmetry for us to use Gauss' law.

(c) **The flux through the top is bigger**. More field lines would go through the top.

(d) The total flux through the cube remains the **same**. The same number of field lines come out. The electric field at each point on the surface becomes **smaller** though – these points are further away from the charge.

Question 3. (10 points) Find the net electric flux through (a) the closed spherical surface in a uniform electric field shown in panel a of the Figure below and (b) the closed cylindrical surface shown in panel b. (c) What can you conclude about the charges, if any, inside the cylindrical surface?

Assume the field is constant in magnitude and with the directions shown. Give your answers in terms of E and R .

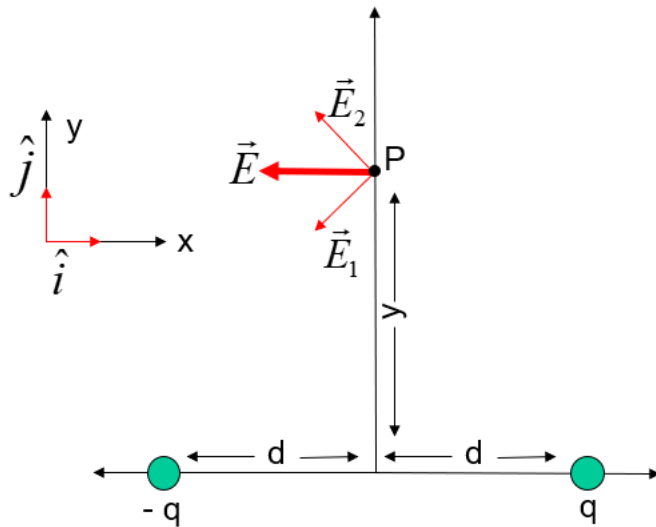


Solutions

- (a) The total flux is **zero**. All the field lines going in also come out
- (b) The flux will be 2X that for a single side and positive and thus equal to $2(EA) = 2E\pi R^2$
- (c) The **net charge is positive** since there are field lines coming out.

Problem 1. (15 points) Two charges of magnitude $q = 2 \text{ nC}$ and $q = -2 \text{ nC}$ are placed on the x-axis a distance $d = 3 \text{ cm}$ from the origin as shown below.

- Find the vector electric field \vec{E} at a point P placed 4 cm from the origin along the y-axis.
- If a $+5 \text{ nC}$ charge were placed at P, what would the force be on this 5 nC charge due to the other two charges?



Solutions.

(a) The electric fields are along the lines connecting each charge to the point P as shown. The magnitudes of these fields are equal as the distances and charge magnitudes are the same. The y-components cancel so the net field is in the negative x-direction.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{E} = 2|\vec{E}_1| \cos(\theta) \hat{i}$$

Where the fact that this is in the negative x-direction would be contained in the angle. Now, just calculating the magnitude we have

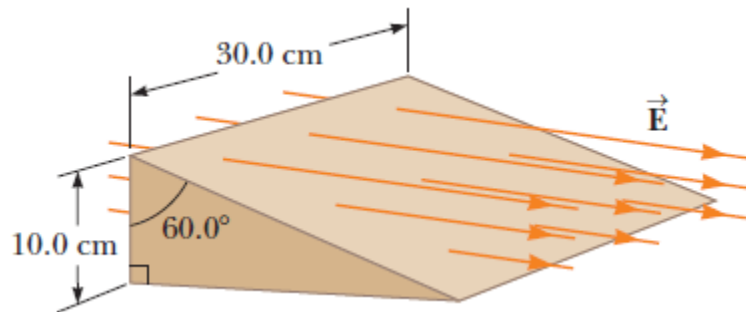
$$E = 2 \frac{kq}{r^2} \cos(\theta) = 2 \frac{kq}{d^2 + y^2} \frac{d}{\sqrt{d^2 + y^2}}$$

$$= 2 \frac{kqd}{(d^2 + y^2)^{3/2}} = 2 \frac{5.4 \times 10^{-1}}{1.25 \times 10^{-4}} = 8.6 \times 10^3 \text{ N/C pointing in the minus x direction.}$$

- Simply apply $\vec{F} = q\vec{E}$. The direction will be the same as the field (positive charge), so towards the negative x-axis. The magnitude is

$$F = (5 \times 10^{-9}) (8.6 \times 10^3) = 4.3 \times 10^{-5} \text{ N}$$

Problem 2. (15 points). Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 6.50 \times 10^4 \text{ N/C}$ as shown in the figure below.



A closed right triangular box with its vertical side on the left and downward slope on the right rests within a horizontal electric field vector \mathbf{E} that points from left to right. The box has a height of 10.0 cm and a depth of 30.0 cm. The downward slope of the box makes an angle of 60 degrees with the vertical.

- Calculate the electric flux through the vertical rectangular surface of the box.
- Calculate the electric flux through the slanted surface of the box.
- Calculate the electric flux through the entire surface of the box.

Solution

a) $\Phi = \mathbf{E} \cdot \mathbf{A} = (6.5 \times 10^4)(0.03)\cos(180) = -1.95 \times 10^3 \text{ Nm}^2/\text{C}$

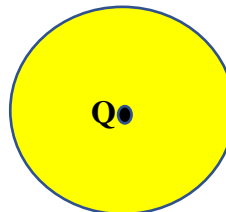
- b) The normal points up from the surface and makes an angle of 60 degrees with the field. The length of the side, L , is given by $\cos(60) = 0.1/L$. $L = 0.2 \text{ m}$.

$$\Phi = \mathbf{E} \cdot \mathbf{A} = (6.5 \times 10^4)(0.2)(0.3)\cos(60) = 1.95 \times 10^3 \text{ Nm}^2/\text{C}$$

(c) The total flux is the sum of that in parts a and b which is zero. We can see that this is the case as the number of field lines going in equal those coming out.

Problem 3. (15 points) A solid insulating sphere of radius 30 cm has a total charge of $9\text{ }\mu\text{C}$ evenly distributed throughout its volume. There is also a positive point charge $Q = +1\text{ }\mu\text{C}$ located at the center of the sphere. Use Gauss' law to calculate the electric field at

- (a) $r = 10\text{ cm}$
- (b) $r = 30\text{ cm}$
- (c) $r = 60\text{ cm}$



Solutions

- (a) We start by drawing a Gaussian sphere of radius 0.1 m centered on the charge Q . The total charge enclosed will be that of charge Q , plus a portion of the charge in the sphere given by the ratio of the volume of the gaussian of radius 0.1 m to that of the actual sphere of radius 0.3 m.

$$\text{So } q_{\text{encl}} = Q + (0.1/0.3)^3 (9 \times 10^{-6}) = 1 \times 10^{-6} + 3.33 \times 10^{-7} = 1.33 \times 10^{-6} \text{ C}$$

The flux is $EA = E(4\pi r^2)$, with $r = 0.1\text{ m}$. Thus the flux is $E(0.126\text{ m}^2)$

From Gauss Law we have $EA = q_{\text{encl}}/\epsilon_0$.

$$E = q_{\text{encl}}/(A\epsilon_0) = (1.33 \times 10^{-6})/[(0.126)(8.85 \times 10^{-12})] = \mathbf{1.19 \times 10^6 \text{ N/C}}$$
 and it points out radially.

- (b) Now we draw the Gaussian surface at $r = 0.3\text{ m}$ and the entire charge is enclosed

$$q_{\text{encl}} = Q + (0.3/0.3)^3 (9 \times 10^{-6}) = 1 \times 10^{-6} + 9 \times 10^{-6} = 1 \times 10^{-5} \text{ C}$$

The flux is $EA = E(4\pi r^2)$, with $r = 0.3\text{ m}$. Thus the flux is $E(1.13\text{ m}^2)$

From Gauss Law we have $EA = q_{\text{encl}}/\epsilon_0$.

$$E = q_{\text{encl}}/(A\epsilon_0) = (1 \times 10^{-5})/[(1.13)(8.85 \times 10^{-12})] = \mathbf{1.0 \times 10^6 \text{ N/C}}$$
 and it points out radially.

- (c) Now we draw the Gaussian surface at $r = 0.6\text{ m}$ and the entire charge is enclosed

The charge enclosed is still $1 \times 10^{-5} \text{ C}$

The flux is $EA = E(4\pi r^2)$, with $r = 0.6\text{ m}$. Thus the flux is $E(4.52\text{ m}^2)$

$$E = q_{\text{encl}}/(A\epsilon_0) = (1 \times 10^{-5})/[(4.52)(8.85 \times 10^{-12})] = \mathbf{2.5 \times 10^5 \text{ N/C}}$$
 and it points out radially.

Possibly Useful Information

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$E = \frac{|q|}{4\pi\epsilon_0 r^2}$$

$$k_e = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} (\text{C}^2 / \text{N}\cdot\text{m}^2)$$

$$\vec{E} = \vec{F} / q_0$$

$$\epsilon_0 \Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$