Prb. # | ANSWERS

1 $F = \frac{k_e \, q_1 q_2}{r^2}$: $k_e \cong 10^{10} \, \text{N/}_{C^2.m^2}$, $r \cong 1 \, m$, q1 = q2 = 1% of charge in person

Say have 100 kg person made of water.

Water is ~ 10 g/mole; 1 mole $\cong 10^{24}$ molecules; water molecules have ~ 10 e.

So in 100 kg have $\frac{10^5 g}{10 g/mole} = 10^4$ moles. Which is 10^{28} atoms, and that gives 10^{29} charges.

1% of this is 10^{27} charges, so $q_1 = q_2 = 10^{27}$ e⁻.

$$F = \frac{10^{10} \ 10^{27} 10^{27} \ 10^{-19} \ 10^{-19}}{1^2} = 10^{26} N$$

Mass of Earth ~ 10²⁵ kg

$$F = mg = 10^{26} N$$

Thus the forces are of the same order of magnitude.

2 (a) Let the third bead have charge *Q* and be located distance *x* from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_{\epsilon}(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_{\epsilon}(q)Q}{(d-x)^2}(-\hat{\mathbf{i}}), \text{ where } d = 1.50 \text{ m}$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d-x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of

$$x = 0.634d = 0.634(1.50 \text{ m}) = \boxed{0.951 \text{ m}}$$

(b) Yes, if the third bead has positive charge. The equilibrium would be stable because if charge *Q* were displaced either to the left or right on the rod, the new net force would be opposite to the direction *Q* has been displaced, causing it to be pushed back to its equilibrium position.

note on (a): This example uses $q_1 = 3q$, but the value of q_1 differs for each person.

Each charge exerts a force of magnitude $\frac{k_e qQ}{(d/2)^2 + x^2}$ on the negative charge -Q: the top charge exerts its force directed upward and to the left, and bottom charge exerts its force directed downward and to the left, each at angle $\theta = \tan^{-1}\left(\frac{d}{2x}\right)$, respectively, above and below the x axis. The two positive charges together exert a net force:

$$\begin{split} \vec{\mathbf{F}} &= -2\frac{k_{e}qQ}{(d/2)^{2} + x^{2}}\cos\theta \hat{\mathbf{i}} \\ &= -2\left[\frac{k_{e}qQ}{(d^{2}/4 + x^{2})}\right]\left[\frac{x}{(d^{2}/4 + x^{2})^{1/2}}\right]\hat{\mathbf{i}} \\ &= \left[\frac{-2xk_{e}qQ}{(d^{2}/4 + x^{2})^{3/2}}\right]\hat{\mathbf{i}} = m\bar{\mathbf{a}} \end{split}$$

or for
$$x \ll \frac{d}{2}$$
, $\vec{\mathbf{a}} \approx -\left(\frac{2k_e qQ}{md^3/8}\right)\vec{\mathbf{x}} \rightarrow \vec{\mathbf{a}} \approx -\left(\frac{16k_e qQ}{md^3}\right)\vec{\mathbf{x}}$

The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in $\vec{a} = -\omega^2 \vec{x}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_e qQ}{3}$.

(b) $\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{16k_eqQ}{md^3} \to T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_eqQ}}}$, where m is the

mass of the object with charge -Q.

(c)
$$v_{\text{max}} = \omega A = 4a\sqrt{\frac{k_e qQ}{md^3}}$$

Call Q = 3.00 nC and q = |-2.00 nC |= 2.00 nC, and r = 4.00 cm = 0.040 0 m. Then,

$$E_1 = E_2 = \frac{k_e Q}{r^2}$$
 and $E_3 = \frac{k_e q}{r^2}$

Then,

$$E_y = 0$$

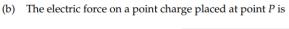
 $E_x = E_{total} = 2 \frac{k_e Q}{r^2} \cos 30.0^\circ - \frac{k_e q}{r^2}$

$$E_x = \frac{k_e}{r^2} \left(2Q \cos 30.0^\circ - q \right)$$

$$E_x = \left[\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{(0.040 \text{ 0 m})^2} \right] \times \left[2(3.00 \times 10^{-9} \text{ C}) \cos 30.0^\circ - 2.00 \times 10^{-9} \text{ C} \right]$$

 $=1.80\times10^{4} \text{ N/C}$

(a)
$$1.80 \times 10^4$$
 N/C to the right



$$F = qE = (-5.00 \times 10^{-9} \text{ C})E = [-8.98 \times 10^{-5} \text{ N (to the left)}]$$

note for (b) that the given value of q may vary

5 Field lines emerge from positive charge and enter negative charge.

(a) The number of field lines emerging from positive q₂ and entering negative charge q₁ is proportional to their charges:

+3.00 nC

ANS. FIG. P22.19

$$\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$$

(b) From above, q_1 is negative, q_2 is positive

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(a)
$$N = \left(\frac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right) \left(6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}\right) \left(47 \frac{\text{electrons}}{\text{atom}}\right)$$

= $\left[2.62 \times 10^{24}\right]$

In (a), the given mass may vary

(b) # electrons added = $\frac{Q}{e}$ = $\frac{1.00 \times 10^{-3} \text{ C added}}{1.60 \times 10^{-19} \text{ C/electron}}$ = 6.25×10^{15} electrons added

Thus,

$$(6.25 \times 10^{15} \text{ added}) \left(\frac{1}{2.62 \times 10^{24} \text{ present}} \right) = \left(\frac{2.38 \text{ added}}{10^9 \text{ present}} \right)$$

→ 2.38 electrons for every 10⁹ already present

In (b), the given value of Q may vary