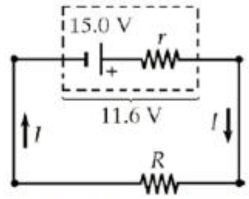
ANSWERS

Note: Given values of certain parameters in this assignment may vary.

1

(a) Combining Joule's law, $P = I\Delta V$, and the definition of resistance, $\Delta V = IR$, gives



$$R = \frac{(\Delta V)^2}{P} = \frac{(11.6\ \text{V})^2}{20.0\ \text{W}} = \boxed{6.73\ \Omega}$$

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V / R$.

$$r = \frac{(\mathcal{E} - IR)}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V} = \frac{(15.0\ \text{V} - 11.6\ \text{V})(6.73\ \Omega)}{11.6\ \text{V}} = \boxed{1.97\ \Omega}$$

2

The equivalent resistance of the parallel combination of three identical resistors is

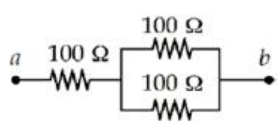
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{R} \quad \text{or} \quad R_p = \frac{R}{3}$$

The total resistance of the series combination between points a and b is then

$$R_{ab} = R + R_p + R = 2R + \frac{R}{3} = \boxed{\frac{7}{3}R}$$

3

(a) Since all the current in the circuit must pass through the series 100-Ω resistor,



$$P_{\max} = I_{\max}^2 R$$

so $$I_{\max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0\ \text{W}}{100\ \Omega}} = 0.500\ \text{A}.$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{eq} I_{\max} = \boxed{75.0\ \text{V}}$$

(b) From a to b in the circuit, the power delivered is

$P_{\text{series}} = \boxed{25.0\ \text{W}}$ for the first resistor, and

$$P_{\text{parallel}} = I^2 R = (0.250\ \text{A})^2(100\ \Omega) = \boxed{6.25\ \text{W}}$$

for each of the two parallel resistors.

(c) $P = I\Delta V = (0.500\ \text{A})(75.0\ \text{V}) = \boxed{37.5\ \text{W}}$

4 If we turn the given diagram on its side and change the lengths of the wires, we find that it is the same as ANS. FIG. P28.9(a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is shown in ANS. FIG. P28.9(b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega} \rightarrow R_{eq} = 2.94\ \Omega$$

This is shown in ANS. FIG. P28.9(c), which in turn reduces to the circuit shown in ANS. FIG. P28.9(d), from which we see that the total resistance of the circuit is 12.94 Ω.

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The total 12.94-Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In ANS. FIG. P28.9(c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

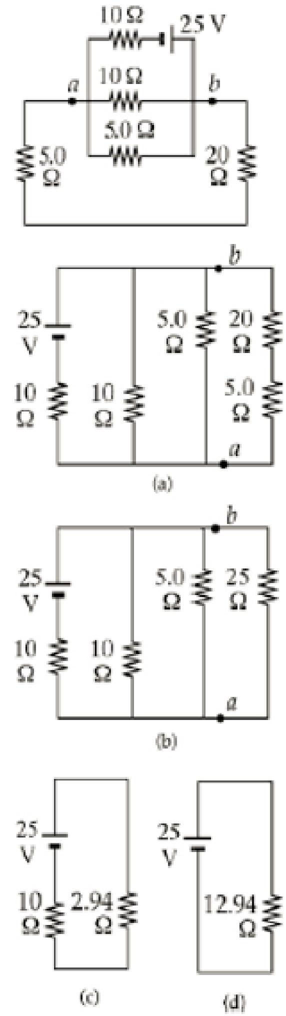
From ANS. FIG. P28.9(b), we see that this potential difference is the same as the potential difference ΔV_{ab} across the 10-Ω resistor and the 5.00-Ω resistor.

Thus we have first found the answer to part (b), which is

$$\Delta V_{ab} = \boxed{5.68\ \text{V}}$$

Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

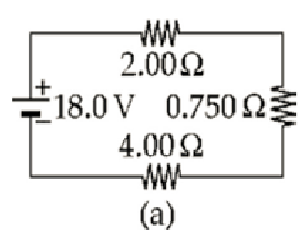
$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$



ANS. FIG. P28.9

5 To find the current in each resistor, we find the resistance seen by the battery. The given circuit reduces as shown in ANS. FIG. P28.19(a), since

$$\frac{1}{(1/1.00\ \Omega) + (1/3.00\ \Omega)} = 0.750\ \Omega$$



In ANS. FIG. P28.19(b),

$$I = 18.0\ \text{V}/6.75\ \Omega = 2.67\ \text{A}$$

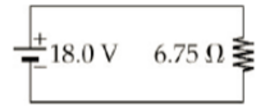
This is also the current in ANS. FIG. P28.19(a), so the 2.00-Ω and 4.00-Ω resistors convert powers

$$P_2 = I\Delta V = I^2R = (2.67\ \text{A})^2(2.00\ \Omega) = \boxed{14.2\ \text{W}}$$

and $P_4 = I^2R = (2.67\ \text{A})^2(4.00\ \Omega) = \boxed{28.4\ \text{W}}$

The voltage across the 0.750-Ω resistor in ANS. FIG. P28.19(a), and across both the 3.00-Ω and the 1.00-Ω resistors in Figure P28.19, is

$$\Delta V = IR = (2.67\ \text{A})(0.750\ \Omega) = \boxed{2.00\ \text{V}}$$



(b)

ANS. FIG. P28.19

Then for the 3.00-Ω resistor,

$$I = \frac{\Delta V}{R} = \frac{2.00\ \text{V}}{3.00\ \Omega}$$

and the power is

$$P_3 = I\Delta V = \left(\frac{2.00\ \text{V}}{3.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{1.33\ \text{W}}$$

For the 1.00-Ω resistor,

$$I = \frac{2.00\ \text{V}}{1.00\ \Omega} \quad \text{and} \quad P_1 = \left(\frac{2.00\ \text{V}}{1.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{4.00\ \text{W}}$$

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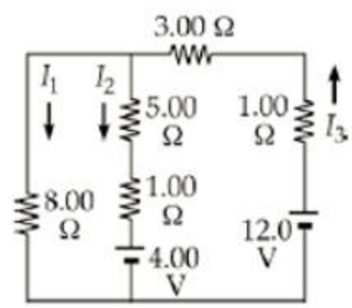
6 We name currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.23. From Kirchhoff's current rule, $I_3 = I_1 + I_2$.



Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\ \text{V} - (4.00\ \Omega)I_3 - (6.00\ \Omega)I_2 - 4.00\ \text{V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00\ \Omega)I_2 - 4.00\ \text{V} + (8.00\ \Omega)I_1 = 0$$

or $\quad (8.00\ \Omega)I_1 = 4.00 + (6.00\ \Omega)I_2$

Solving the above linear system (by substituting $I_1 + I_2$ for I_3), we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = \frac{4}{3}I_1 - \frac{2}{3} \end{cases}$$

and to the single equation

$$8 = 4I_1 + 10\left(\frac{4}{3}I_1 - \frac{2}{3}\right) = \frac{52}{3}I_1 - \frac{20}{3}$$

which gives

$$I_1 = \frac{3}{52}\left(8 + \frac{20}{3}\right) = 0.846\ \text{A}$$

Then $\quad I_2 = I_2 = \frac{4}{3}(0.846) - \frac{2}{3} = 0.462$

and $\quad I_3 = I_1 + I_2 = 1.31\ \text{A}$

give $\quad \boxed{I_1 = 846\ \text{mA},\ I_2 = 462\ \text{mA},\ I_3 = 1.31\ \text{A}}$

(a) The results are: 0.846 A down in the 8.00-Ω resistor; 0.462 A down in the middle branch; 1.31 A up in the right-hand branch.

(b) For 4.00-V battery:

$$\Delta U = P\Delta t = (\Delta V)I\Delta t = (4.00\ \text{V})(-0.462\ \text{A})(120\ \text{s}) = -222\ \text{J}$$

For 12.0-V battery:

$$\Delta U = (12.0\ \text{V})(1.31\ \text{A})(120\ \text{s}) = 1.88\ \text{kJ}$$

The results are: $\boxed{-222\text{ J by the 4.00-V battery and 1.88 kJ by the 12.0-V battery}}$.

(c) To the 8.00-Ω resistor:

$$\Delta U = I^2 R\Delta t = (0.846\ \text{A})^2(8.00\ \Omega)(120\ \text{s}) = \boxed{687\ \text{J}}$$

To the 5.00-Ω resistor:

$$\Delta U = (0.462\ \text{A})^2(5.00\ \Omega)(120\ \text{s}) = \boxed{128\ \text{J}}$$

To the 1.00-Ω resistor in the center branch:

$$(0.462\ \text{A})^2(1.00\ \Omega)(120\ \text{s}) = \boxed{25.6\ \text{J}}$$

To the 3.00-Ω resistor:

$$(1.31\ \text{A})^2(3.00\ \Omega)(120\ \text{s}) = \boxed{616\ \text{J}}$$

To the 1.00-Ω resistor in the right-hand branch:

$$(1.31\ \text{A})^2(1.00\ \Omega)(120\ \text{s}) = \boxed{205\ \text{J}}$$

(d) Chemical energy in the 12.0-V battery is transformed into internal energy in the resistors. The 4.00-V battery is being charged, so its chemical potential energy is increasing at the expense of some of the chemical potential energy in the 12.0-V battery.

(e) Either sum the results in part (b): $-222\ \text{J} + 1.88\ \text{kJ} = 1.66\ \text{kJ}$,

or in part (c): $687\ \text{J} + 128\ \text{J} + 25.6\ \text{J} + 616\ \text{J} + 205\ \text{J} = 1.66\ \text{kJ}$

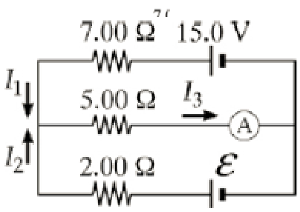
The total amount of energy transformed is $\boxed{1.66\ \text{kJ}}$

7

(a) For the upper loop:

$$+15.0\ \text{V} - (7.00\ \Omega)I_1 - (2.00\ \text{A})(5.00\ \Omega) = 0$$

$$5.00 = 7.00 I_1 \quad \text{so} \quad \boxed{I_1 = 0.714\ \text{A}}$$



(b) For the center-left junction:

$$I_3 = I_1 + I_2 = 2.00\ \text{A}$$

where I_3 is the current through the ammeter (assumed to travel to the right):

$$0.714 + I_2 = 2.00 \quad \text{so} \quad \boxed{I_2 = 1.29\ \text{A}}$$

(c) For the lower loop:

$$+\mathcal{E} - (2.00\ \Omega)(1.29\ \text{A}) - (5.00\ \Omega)(2.00\ \text{A}) = 0 \rightarrow \boxed{\mathcal{E} = 12.6\ \text{V}}$$

(d) Kirchhoff's junction rule for the circuit (see the figure below) gives

$$I_3 = I_1 + I_2 = 1.19\ \text{A}$$

For the top loop, Kirchhoff's loop rule gives

$$+15.0\ \text{V} - (7.00\ \Omega)I_1 - (5.00\ \Omega)I_3 = 0$$
$$+15.0\ \text{V} - (7.00\ \Omega)I_1 - (5.00\ \Omega)(1.19\ \text{A}) = 0$$

Solving for I_1, we obtain

$$(7.00\ \Omega)I_1 = +15.0\ \text{V} - (5.00\ \Omega)(1.19\ \text{A}) \rightarrow I_1 = 1.29\ \text{A}$$

Then, $I_2 = I_3 - I_1 = 1.19\ \text{A} - 1.29\ \text{A} = -0.103\ \text{A}$. Applying Kirchhoff's loop rule to the bottom loop, we obtain

$$+\mathcal{E} - (2.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$$
$$\mathcal{E} = (2.00\ \Omega)I_2 + (5.00\ \Omega)I_3 = (2.00\ \Omega)(-0.103\ \text{A}) + (5.00\ \Omega)(1.19\ \text{A})$$
$$\mathcal{E} = 5.74\ \text{V}$$

8

Label the currents in the branches as shown in ANS. FIG. P28.27(a). Reduce the circuit by combining the two parallel resistors as shown in ANS. FIG. P28.27(b).

Apply Kirchhoff's loop rule to both loops in ANS. FIG. P28.27(b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250\ \text{V}$$

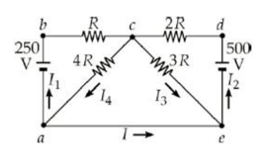
$$(1.71R)I_1 + (3.71R)I_2 = 500\ \text{V}$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

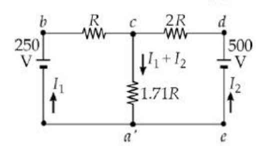
$$I_1 = 10.0\ \text{mA}$$

$$I_2 = 130.0\ \text{mA}$$

From ANS. FIG. P28.27(b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240\ \text{V}$.



ANS. FIG. P28.27(a)



ANS. FIG. P28.27(b)

Thus, from ANS. FIG. P28.27(a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\ \text{V}}{4\,000\ \Omega} = 60.0\ \text{mA}$.

Finally, applying Kirchhoff's point rule at point a in ANS. FIG. P28.27(a) gives:

$$I = I_4 - I_1 = 60.0\ \text{mA} - 10.0\ \text{mA} = +50.0\ \text{mA}$$

or $\quad I = \boxed{50.0\ \text{mA from point } a \text{ to point } e}$.

9	**(a) The time constant of the circuit is** $$\tau = RC = (100\ \Omega)(20.0\times10^{-6}\ \text{F}) = 2.00\times10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$$ (b) The maximum charge on the capacitor is given by Equation 28.13: $$Q_{\max} = C\mathcal{E} = (20.0\times10^{-6}\ \text{F})(9.00\ \text{V}) = \boxed{1.80\times10^{-4}\ \text{C}}$$ (c) We use $q(t) = Q_{\max}\left(1-e^{-t/RC}\right)$, when $t = RC$. Then, $$q(t) = Q_{\max}\left(1-e^{-RC/RC}\right) = Q_{\max}\left(1-e^{-1}\right) = (1.80\times10^{-4}\ \text{C})\left(1-e^{-1}\right) = \boxed{1.14\times10^{-4}\ \text{C}}$$
10	The potential difference across the capacitor is $$\Delta V(t) = \Delta V_{\max}\left(1-e^{-t/RC}\right)$$ Using 1 farad = 1 s/Ω, $$4.00\ \text{V} = (10.0\ \text{V})\left[1-e^{-(3.00\ \text{s})/\left[R(10.0\times10^{-6}\ \text{s}/\Omega)\right]}\right]$$ Therefore, $$0.400 = 1.00 - e^{-(3.00\times10^{5}\ \Omega)/R}$$ or $$e^{-(3.00\times10^{5}\ \Omega)/R} = 0.600.$$ Taking the natural logarithm of both sides, $$-\frac{3.00\times10^{5}\ \Omega}{R} = \ln(0.600)$$ and $$R = -\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)} = +5.87\times10^{5}\ \Omega = \boxed{587\ \text{k}\Omega}.$$