PHY 114. HW Chapter 29 Solutions. Spring 2023

ANSWERS

1

Imagine grasping the conductor with the right hand so the fingers curl around the conductor in the direction of the magnetic field. The thumb then points along the conductor in the direction of the current. The results are

(a) [toward the left] (b) [out of the page] (c) [lower left to upper right]

2

Note: Given values may vary throughout the assignment.

The magnetic field is given by

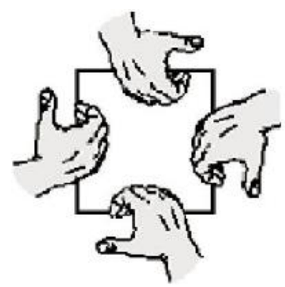
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(2.00\ \mathrm{A})}{2\pi(0.250\ \mathrm{m})} = \boxed{1.60\times10^{-6}\ \mathrm{T}}$$

3

(a) Use Equation 30.4 for the field produced by each side of the square.

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

where $\theta_1 = 45.0°$, $\theta_2 = -45.0°$, and $a = \frac{\ell}{2}$



Each side produces a field into the page. The four sides altogether produce

$$B_{\text{center}} = 4B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\mu_0 I}{\pi\ell/2}[\sin 45.0° - \sin(-45.0°)]$$

$$= \frac{2\mu_0 I}{\pi\ell}\left[\frac{2}{\sqrt{2}}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi\ell}$$

$$B = \frac{2\sqrt{2}(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10.0\ \mathrm{A})}{\pi(0.400\ \mathrm{m})}$$

$$= 2\sqrt{2}\times10^{-5}\ \mathrm{T} = \boxed{28.3\ \mu\mathrm{T\ into\ the\ page}}$$

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10.0\ \mathrm{A})}{4(0.400\ \mathrm{m})}$$

$$= \boxed{24.7\ \mu\mathrm{T\ into\ the\ page}}$$

4

We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{\mathbf{s}}$ is at 0° or 180° to $\hat{\mathbf{r}}$, so $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = 0$. Thus, only the curved section of wire contributes to $\vec{\mathbf{B}}$ at P. Hence, $d\vec{\mathbf{s}}$ is tangent to the arc and $\hat{\mathbf{r}}$ is radially inward; so $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = |ds|1\sin 90°\otimes = |ds|\otimes$. All points along the curve are the same distance $r = 0.600$ m from the field point, so

$$B = \int_{\text{all current}} |d\vec{\mathbf{B}}| = \int \frac{\mu_0}{4\pi}\frac{I|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int|ds| = \frac{\mu_0}{4\pi}\frac{I}{r^2}s$$

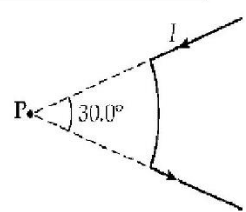
where s is the arc length of the curved wire,

$$s = r\theta = (0.600\ \mathrm{m})(30.0°)\left(\frac{2\pi}{360°}\right) = 0.314\ \mathrm{m}$$

Then,

$$B = (10^{-7}\ \mathrm{T\cdot m/A})\frac{(3.00\ \mathrm{A})}{(0.600\ \mathrm{m})^2}(0.314\ \mathrm{m})$$

$$B = \boxed{262\ \mathrm{nT\ \ into\ the\ page}}$$

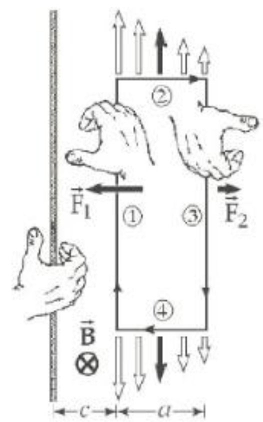


5 (a) From Equation 30.12, the force per unit length that one wire exerts on the other is $F/\ell = \mu_0 I_1 I_2 / 2\pi d$, where d is the distance separating the two wires. In this case, the value of this force is

$$\frac{F}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)\left(3.00\ \mathrm{A}\right)^2}{2\pi\left(6.00 \times 10^{-2}\ \mathrm{m}\right)} = \boxed{3.00 \times 10^{-5}\ \mathrm{N/m}}$$

(b) We can answer this question by consulting Section 30.2 in the textbook, or we can reason it out. Imagine these two wires lying side by side on a table with the two currents flowing toward you, wire 1 on the left and wire 2 on the right. The right-hand rule that relates current to field direction shows the magnetic field due to wire 1 at the location of wire 2 is directed vertically upward. Then, the right-hand rule that relates current and field to force gives the direction of the force experienced by wire 2, with its current flowing through this field, as being to the left, back toward wire 1. Thus, the force one wire exerts on the other is an $\boxed{\text{attractive}}$ force.

6 To the right of the long, straight wire, current I_1 creates a magnetic field into the page. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (from Equation 30.12):



$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{-a}{c(c+a)}\right]\hat{\mathbf{i}}$$

$$\vec{F} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)(5.00\ \mathrm{A})(10.0\ \mathrm{A})(0.450\ \mathrm{m})}{2\pi} \times \left(\frac{-0.150\ \mathrm{m}}{(0.100\ \mathrm{m})(0.250\ \mathrm{m})}\right)\hat{\mathbf{i}}$$

$$\vec{F} = \left(-2.70 \times 10^{-5}\hat{\mathbf{i}}\right)\ \mathrm{N} = \left(-27.0 \times 10^{-6}\hat{\mathbf{i}}\right)\ \mathrm{N} = \boxed{-27.0\hat{\mathbf{i}}\ \mu\mathrm{N}}$$ (to the right).

7 (a) From Ampère's law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(1.00\ \mathrm{A})}{2\pi\left(1.00 \times 10^{-3}\ \mathrm{m}\right)} = \boxed{200\ \mu\mathrm{T}\ \text{toward top of page}}$$

(b) Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, $I_b = 1.00\ \mathrm{A} - 3.00\ \mathrm{A} = -2.00$ A, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi\left(3.00 \times 10^{-3}\ \mathrm{m}\right)} = \boxed{133\ \mu\mathrm{T}\ \text{toward bottom of page}}$$

8	The magnetic field at the center of a solenoid is $B=\mu_0 \frac{N}{\ell} I$, so $$I=\frac{B}{\mu_0 n}=\frac{(1.00\times10^{-4}\text{ T})(0.400\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1\,000)}=\boxed{31.8\text{ mA}}$$
9	(a) The magnetic flux through the flat surface S_1 is $$(\Phi_B)_{\text{flat}}=\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}=B\pi R^2\cos(180-\theta)=\boxed{-B\pi R^2\cos\theta}$$ (b) The net flux out of the closed surface is zero: $$(\Phi_B)_{\text{flat}}+(\Phi_B)_{\text{curved}}=0$$ Therefore, $$(\Phi_B)_{\text{curved}}=\boxed{B\pi R^2\cos\theta}$$

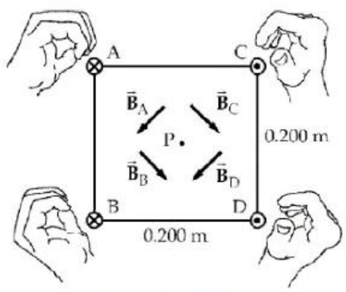
10

Each wire is distant from P by ($\ell=0.200$ m)

$$r=\sqrt{\ell^2+\ell^2}/2=\ell/\sqrt{2}$$

and each wire produces a field at P of equal magnitude

$$B=\frac{\mu_0 I}{2\pi r}$$



ANS. FIG. P30.39

Carrying currents into the page, A produces at P a field to the left and downward at –135°, while B creates a field to the right and downward at –45°. Carrying currents out of the page, C produces a field downward and to the right at –45°, while D's contribution is downward and to the left. All horizontal components cancel; thus, all remaining components are vertically downward. The magnitude of the resulting field is

$$B_P=4B\cos 45.0°=4\frac{\mu_0 I}{2\pi r}\cos 45.0°=4\frac{\mu_0 I}{2\pi(\ell/\sqrt{2})}\frac{1}{\sqrt{2}}=\frac{2\mu_0 I}{\pi\ell}$$

$$=\frac{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\text{ A})}{\pi(0.200\text{ m})}=2.00\times10^{-5}\text{ T}$$

The magnetic field is $\boxed{20.0\ \mu\text{T toward the bottom of the page}}$.

(c) Solution to **What If?**

From part (a) and part (b) of the problem,

$$\vec{\mathbf{B}}_P=(-1.44\times10^{-5}\text{ T})\hat{\mathbf{j}}$$

where $+x$ is to the right, $+y$ is upward, and $+z$ is out of the page. The magnetic force on a charged particle is given as

$$\vec{\mathbf{F}}_B=q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

with $\vec{\mathbf{v}}=(-3.33\times10^{5}\text{ m/s})\hat{\mathbf{k}}$ The magnetic force on the electron is then

$$\vec{\mathbf{F}}_B=-evB(-\hat{\mathbf{k}}\times-\hat{\mathbf{j}})=evB\hat{\mathbf{i}}$$

and, from Newton's second law, the acceleration of the electron is

$$\vec{\mathbf{a}}=\frac{\vec{\mathbf{F}}_B}{m_e}=\frac{evB}{m_e}\hat{\mathbf{i}}$$

Substituting numerical values,

$$\vec{\mathbf{a}}=\left[\frac{(1.60\times10^{-19}\text{ C})(3.33\times10^{5}\text{ m/s})(1.44\times10^{-5}\text{ T})}{(9.11\times10^{-31}\text{ kg})}\right]\hat{\mathbf{i}}$$

$$=(8.42\times10^{11}\text{ m/s}^2)\hat{\mathbf{i}}\text{ (to the right)}$$