ANSWERS

Note: Certain given values may vary.

1

From Equation 30.2,

$$\mathcal{E} = -N\frac{\Delta(BA\cos\theta)}{\Delta t} = -NB\pi r^2\left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$$

$$= -25.0\left(50.0\times10^{-6}\ \mathrm{T}\right)\left[\pi(0.500\ \mathrm{m})^2\right]\left(\frac{\cos180° - \cos0°}{0.200\ \mathrm{s}}\right)$$

$$\mathcal{E} = \boxed{+9.82\ \mathrm{mV}}$$

2

Faradays law gives

$$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N\left[\frac{d}{dt}\left(0.010\,0t + 0.040\,0t^2\right)\right]A$$

or $$|\mathcal{E}| = N(0.010\,0 + 0.080\,0t)A$$

where $|\mathcal{E}|$ is in volts, A is in meters squared, and t is in seconds. At $t = 5.00$ s, suppressing units,

$$|\mathcal{E}| = 30.0[0.010\ 0 + 0.080\ 0(5.00)]\left[\pi(0.040\ 0)^2\right]$$

$$= 6.18\times10^{-2} = \boxed{61.8\ \mathrm{mV}}$$

3

(a) At a distance x from the long, straight wire, the magnetic field is $B = \frac{\mu_0 I}{2\pi x}$. The flux through a small rectangular element of length L and width dx within the loop is

$$d\Phi_B = \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi x}L\,dx:$$

$$\Phi_B = \int_h^{h+w}\frac{\mu_0 IL}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)}$$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

where $\frac{dI}{dt} = \frac{d}{dt}(a+bt) = b$:

$$\mathcal{E} = -\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(1.00\ \mathrm{m})}{2\pi}\times\ln\left(\frac{0.010\ 0\ \mathrm{m} + 0.100\ \mathrm{m}}{0.010\ 0\ \mathrm{m}}\right)(10.0\ \mathrm{A/s})$$

$$= -4.80\times10^{-6}\ \mathrm{V}$$

(c) The long, straight wire produces magnetic flux into the page through the rectangle, shown in ANS. FIG. P31.13. As the magnetic flux increases, the rectangle produces its own magnetic field out of the page to oppose the increase in flux. The induced current creates this opposing field by traveling $\boxed{\text{counterclockwise}}$ around the loop.

(d) From part (a) of the problem, the magnetic flux through the rectangular loop is given by

$$\Phi_B = \frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)$$

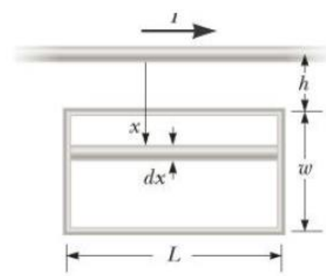
In this case, I is constant, but $h = h_0 + vt$. The emf induced in the loop is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left(\frac{\mu_0 IL}{2\pi}\right)\left(\frac{h}{h+w}\right)\frac{d}{dt}\left(1+\frac{w}{h}\right)$$

$$= -\frac{\mu_0 ILwv}{2\pi}\left(\frac{h}{h+w}\right)\left(-\frac{1}{h^2}\right) = \frac{\mu_0 ILw}{2\pi h(h+w)}$$

Substituting for h gives

$$\mathcal{E} = \frac{\mu_0 ILwv}{2\pi(h_0+vt)(h_0+vt+w)}$$



ANS. FIG. P31.13

(e) As the loop moves away from the straight wire, the magnetic flux within the loop, which is into the page, decreases in strength. Therefore, the induced current must produce magnetic flux into of the page to oppose the decrease in magnetic flux. Therefore, the current must be in the clockwise direction.

4

(a) $\vec{\mathbf{B}}_{ext} = B_{ext}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{induced} = B_{induced}\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed from a to b, $\boxed{\text{to the right}}$.

(b) $\vec{\mathbf{B}}_{ext} = B_{ext}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\vec{\mathbf{B}}_{induced} = B_{induced}\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed from a to b, $\boxed{\text{out of the page}}$ in the textbook picture.

(c) $\vec{\mathbf{B}}_{ext} = B_{ext}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{induced} = B_{induced}\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed from a to b, $\boxed{\text{to the right}}$.

5

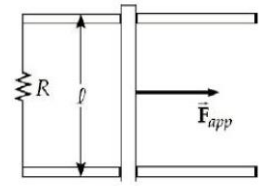
(a) Refer to ANS. FIG. P31.26 above. At constant speed, the net force on the moving bar equals zero, or

$$\left|\vec{\mathbf{F}}_{\text{app}}\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right|$$

where the current in the bar is $I = \mathcal{E}/R$ and the motional emf is $\mathcal{E} = B\ell v$. Therefore,

$$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50\ \text{T})^2(1.20\ \text{m})^2(2.00\ \text{m/s})}{6.00\ \Omega}$$
$$= 3.00\ \text{N}$$

The applied force is $\boxed{3.00\ \text{N to the right}}$.



ANS. FIG. P31.26

(b) $P = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00\ \text{W}$ or $P = Fv = \boxed{6.00\ \text{W}}$

6

(a) The emfs induced in the rods are proportional to the lengths of the sections of the rods between the rails. The emfs are $\mathcal{E}_1 = B\ell v_1$ with positive end downward, and $\mathcal{E}_2 = B\ell v_2$ with positive end upward, where $\ell = d = 10.0$ cm is the distance between the rails.

We apply Kirchhoff's laws. We assume current I_1 travels downward in the left rod, current I_2 travels upward in the right rod, and current I_3 travels upward in the resisitor R_3.

For the left loop, $+B\ell v_1 - I_1R_1 - I_3R_3 = 0$ **[1]**

For the right loop, $+B\ell v_2 - I_2R_2 + I_3R_3 = 0$ **[2]**

At the top junction, $I_1 = I_2 + I_3$ **[3]**

Substituting [3] into [1] gives

$$B\ell v_1 - I_1R_1 - I_3R_3 = 0$$
$$B\ell v_1 - (I_2 + I_3)R_1 - I_3R_3 = 0$$
$$I_2R_1 + I_3(R_1 + R_3) = B\ell v_1 \quad \textbf{[4]}$$

Now using [2] and [4] to solve for I_2,

$$I_2 = \frac{B\ell v_2 + I_3R_3}{R_2} = \frac{B\ell v_1 - I_3(R_1 + R_3)}{R_1}$$

then equating gives

$$(B\ell v_2 + I_3R_3)R_1 = [B\ell v_1 - I_3(R_1 + R_3)]R_2$$
$$I_3[R_3R_1 + (R_1 + R_3)R_2] = B\ell v_1R_2 - B\ell v_2R_1$$

Solving for I3 gives

$$I_3 = B\ell\frac{(v_1R_2 - v_2R_1)}{R_1R_2 + R_1R_3 + R_2R_3}$$

Substituting numerical values, and noting that

$$R_1R_2 + R_1R_3 + R_2R_3 = (10.0\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(5.00\ \Omega) + (15.0\ \Omega)(5.00\ \Omega) = 275\ \Omega^2$$

we obtain

$$I_3 = (0.010\ 0\ \text{T})(0.100\ \text{m}) \times \frac{[(4.00\ \text{m/s})(15.0\ \Omega) - (2.00\ \text{m/s})(10.0\ \Omega)]}{275\ \Omega^2}$$
$$= 1.45 \times 10^{-4}\ \text{A}$$

Therefore, $I_3 = \boxed{145\ \mu\text{A upward in the picture}}$, as was originally chosen.

(b) Changing the directions of the rods changes the polarity, but not the magnitude, of each emf.

Note that we reversed the expected directions of the currents. Again we can apply Kirchhoff's laws.

For the left loop (going clockwise)

$$|\mathcal{E}_1| - I_3R_3 - I_1R_1 = 0.$$

For the right loop (going clockwise)

$$|\mathcal{E}_2| - I_2R_2 + I_3R_3 = 0.$$

And for the bottom junction

$$I_1 = I_3 + I_2.$$

Note these equations are identical to equations (3), (4), and (5) from part (a), so the result will be the same as well. Therefore, the magnitude of the current will be the same. And, since we reversed the direction of the current in the circuit diagram, and the result is again positive, the direction of the current is downward.

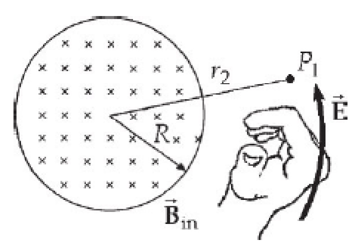
7

Point P_1 lies outside the region of the uniform magnetic field. The rate of change of the field, in teslas per second, is

$$\frac{dB}{dt} = \frac{d}{dt}(2.00t^3 - 4.00t^2 + 0.800) = 6.00t^2 - 8.00t$$

where t is in seconds. At $t = 2.00$ s, we see that the field is increasing:

$$\frac{dB}{dt} = 6.00(2.00)^2 - 8.00(2.00) = 8.00\ \text{T/s}$$



The magnetic flux is increasing into the page; therefore, by the right-hand rule (see figure), the induced electric field lines are counter-clockwise. [Also, if a conductor of radius r_1 were placed concentric with the field region, by Lenz's law, the induced current would be counterclockwise. Therefore, the direction of the induced electric field lines are counterclockwise.] The electric field at point P_1 is tangent to the electric field line passing through it.

(a) The magnitude of the electric field is

$$|\text{E}| = \frac{\text{R}^2}{2\ \text{r}}\frac{\text{dB}}{\text{dt}} = \frac{(2.25\ \times\ 10^{-2})^2}{2\ (0.051)}\ (12.00\text{t}^2 - 2.00\text{t}) = 0.27\frac{\text{N}}{\text{C}}$$

$$\text{F} = \text{qE} = (1.6 \times\ 10^{-19}\ \text{C})\left(0.27\frac{\text{N}}{\text{C}}\right) = 4.32 \times\ 10^{-20}\ \text{N}$$

(b) Because the electron holds a negative charge, the direction of the force is opposite to the field direction. The force is $\boxed{\text{tangent to the electric field line passing through at point } P_1 \text{ and clockwise.}}$

(c) The force is zero when the rate of change of the magnetic field is zero:

$$\frac{dB}{dt} = 6.00t^2 - 8.00t = 0 \rightarrow t = \boxed{0} \text{ or } t = \frac{8.00}{6.00} = \boxed{1.33\ \text{s}}$$

8	(a) Use Equation 31.11, where B is the horizontal component of the magnetic field because the coil rotates about a vertical axis: $$\mathcal{E}_{max} = NB_{horizontal}A\omega = 100(2.00\times10^{-5}\ \text{T})(0.200\ \text{m})^2 \times\left[\left(1500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right] = 1.26\times10^{-2}\ \text{V} = \boxed{12.6\ \text{mV}}$$ (b) Maximum emf occurs when the magnetic flux through the coil is changing the fastest. This occurs at the moment when the flux is zero, which is when the plane of the coil is parallel to the magnetic field.