

# Physics 114

## Exam 3 Spring 2023

### Solutions

Name: \_\_\_\_\_

For grading purposes (do not write here):

Question

Problem

1.

1.

2.

2.

3.

3.

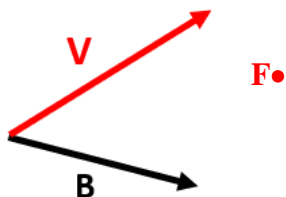
**Include a statement and signature that you have followed the honor code and not cheated on this exam.**

Answer each of the following questions and each of the problems. Points for each question and problem are indicated in red with the amount being spread equally among parts (a,b,c etc) unless otherwise noted. Be sure to show all your work. Use the back of the pages if necessary.

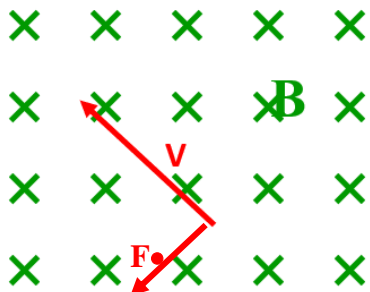
Question 1. (10 points)

- (a) (4 points) Determine the direction of the force on a moving charge as described below. Draw the force or clearly indicate its precise direction.

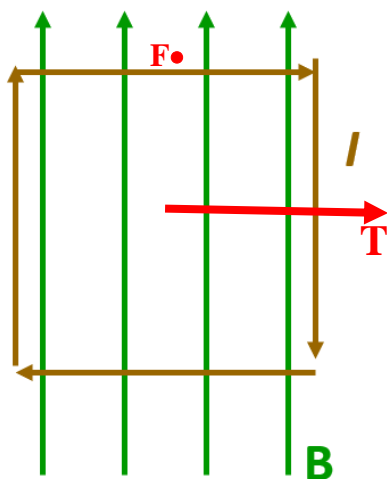
- (i) A negative charge moving in a magnetic field: **The Force is out of the page. Note the charge is negative**



- (ii) A positive charge moving in a magnetic field that is directed into the page. **The force is down to the left as shown**



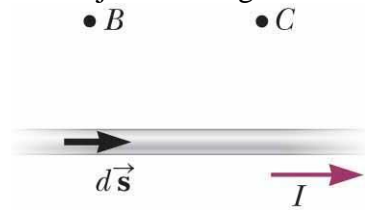
- (b) (6 points) A current carrying loop is placed in a uniform magnetic field as shown below.



- (i) Is there a force **on the top wire**? If so, describe and draw it. (Show its direction). **Yes, the force is given by  $\mathbf{F} = \mathbf{I}\mathbf{L} \times \mathbf{B}$  and is out of the page.**
- (ii) Is there a net force on the entire loop? If so, describe and draw it. **No, the net magnetic force on a current carrying loop in a uniform magnetic field is always zero.**
- (iii) Is there a net torque on the loop? If so, describe and draw it. **Yes. The torque is given by  $\boldsymbol{\mu} \times \mathbf{B}$  and the magnetic moment,  $\boldsymbol{\mu}$ , is into the page. The torque is to the right.**

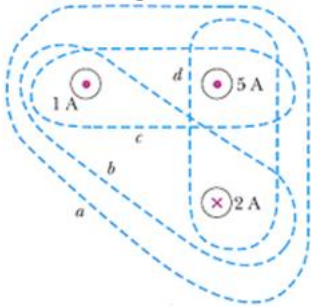
Question 2. (10 points).

- (a) Consider the magnetic field due to the current in the wire shown in the figure. Rank the points  $A$ ,  $B$ , and  $C$  in terms of magnitude of the magnetic field that is due to the current in just the length element  $ds$  shown from greatest to least.



The rank is B, C, A as seen by evaluating the integrand in Biot Savart's law.

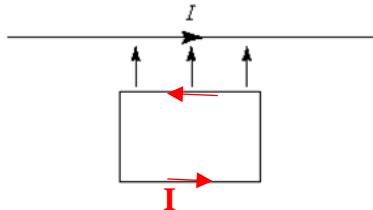
- (b) Rank the magnitudes of  $\oint \mathbf{B} \cdot d\mathbf{s}$  for the closed paths in the figure, from greatest to least.



The rank is C, A, D, B, given by the sum of the currents penetrating the areas enclosed by each loop.

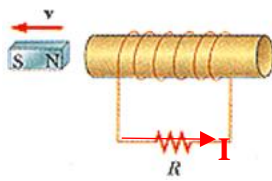
Question 3. (10 points)

- (a) A long, straight wire carries a steady current  $I$ . A rectangular conducting loop lies in the same plane as the wire, with two sides parallel to the wire and two sides perpendicular. Suppose the loop is pushed toward the wire as shown. Is there an induced current in the loop? If so, is it clockwise or counter clockwise (draw it on the loop).



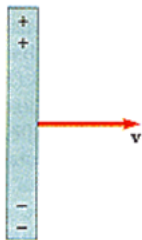
Yes, there is a current induced in the loop. The flux through the loop due to the magnetic field created by the current is into the page. It is increasing as the loop approaches the wire. Thus, there is an induced current in the loop that wants to create flux out of the page. Thus is counter clockwise.

- (b) What is the direction of the induced current in resistor  $R$  in the figure below when the bar magnet is moved to the left? (Draw it)



The flux of the magnet is to the right. It is decreasing. The induced current wants to put that flux back in to the right. The current would go from left to right.

- (c) A copper bar is moved to the right, as shown below, while its axis is maintained perpendicularly to a large, uniform magnetic field. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?



The magnetic force on the positive charges is upward. We are told that  $v$  is perpendicular to  $B$ . Thus the magnetic field must be into the page.

Problem 1. (15 points) A particle that has a mass of 0.02 Kg has a charge of 6 C and a velocity given by  $\vec{v} = (5\hat{i} + 3\hat{j})$  m/s. It is travelling in a magnetic field given by  $\vec{B} = (3\hat{i} + 3\hat{k})$  T.

(a) (10 points) Determine the magnetic force on the charge.

$$\text{The force is given by } q\mathbf{v} \times \mathbf{B} = 6(5\hat{i} + 3\hat{j}) \times (3\hat{i} + 3\hat{k}) = 6(15(\hat{i} \times \hat{i}) + 15(\hat{i} \times \hat{k}) + 9(\hat{j} \times \hat{i}) + 9(\hat{j} \times \hat{k})) = 6(-15\hat{j} - 9\hat{k} + 9\hat{i}) = 54\hat{i} - 90\hat{j} - 54\hat{k}$$

(b) (3 points) What is the angle between the velocity and the magnetic field?

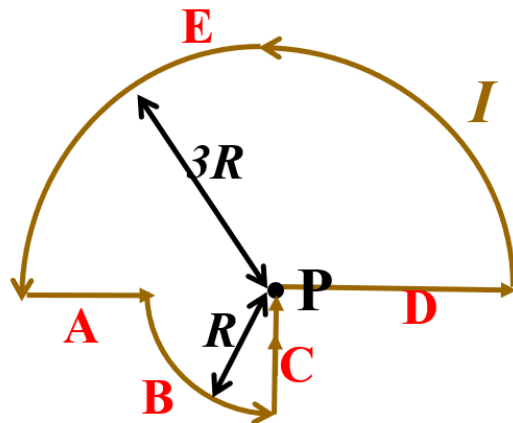
We have

$|\vec{F}| = qvB\sin(\theta)$ , so we need to calculate the magnitude of  $\mathbf{F}$ ,  $\mathbf{v}$  and  $\mathbf{B}$  by taking the square root of the sum of the components. The magnitudes are  $F = 118$ ,  $v = 5.83$ , and  $B = 4.24$ . So  $\sin(\theta) = 118/(6 \cdot 5.83 \cdot 4.24) = 0.795$  which gives  $\theta = 52.7^\circ$ .

(c) (2 points) What is the angle between the force and the velocity?

The force and the velocity are always perpendicular to each other, so 90 degrees.

Problem 2. (15 points). A current of 4 A is carried by wires as shown below where section A points directly at point P and then the wire forms a  $\frac{1}{4}$  circle centered at point P with radius  $R = 1$  m (section B), then section C takes the



current directly to point P where it turns and section D takes it away towards the semicircular loop E of radius  $3R$  centered on point P.

(a) (3 points) Can Ampere's law be used to calculate the magnetic field at point P? Why or why not?

No, there is not enough symmetry

(b) (4 points) Find the direction of the magnetic field at point P

Using either the loop right hand rule or the wire one, we see that the magnetic field is upward out of the page.

(c) (8 points) Find the magnitude of the magnetic field at point P.

The straight wire segments A, C and D do not contribute as  $\vec{ds} \times \hat{r} = 0$ .

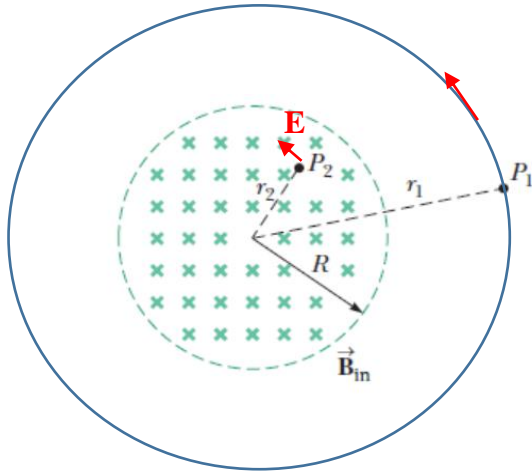
For segment B,  $B = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{s}{R^2} = \frac{\mu_0 I}{4\pi} \frac{2\pi R}{4R^2} = \frac{\mu_0 I}{8R} = (1.26\text{E-}6)(4)/(8)(1) = 6.28\text{E-}7$

For segment E, we follow the same calculation but  $s = 2\pi r/2$  as it is a semicircle.

$B = \frac{\mu_0 I}{4\pi} \frac{2\pi(3R)}{2(3R)^2} = \frac{\mu_0 I}{12R} = (1.26\text{E-}6)(4)/(12)(1) = 4.19\text{E-}7$

So the total field is the sum of these which is  $1.05 \text{ E-}6 \text{ T}$

**Problem 3. (15 points).** Within the green dashed circle of radius  $R = 2$  cm shown in the figure below, the magnetic field increases at a constant rate of 8 Teslas/second (so the field is like that of an ideal solenoid pointing into the page). There is a wire concentric with the magnetic field of radius  $r_1 = 3$  cm.



be counter clockwise.

(a) (7 points). Calculate the emf induced in the wire and describe the direction of the induced current (draw it).

$$\varepsilon = \oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt}$$

The flux  $\Phi_B$  is just  $(B)(A)$  where  $A$  is the area that has flux through it. The area is  $\pi R^2$

$$\text{So we have } \frac{d\Phi_B}{dt} = A \frac{dB}{dt} = (1.26\text{E-}3)(8) = 0.01 \text{ Wb/s}$$

$$\varepsilon = 10 \text{ mV}$$

The flux is increasing into the page so we want an induced field coming out of the page. The current will

(b) (4 points). What is the rate of change of magnetic flux through the circle of radius  $r_2 = 1.5$  cm?

We still have  $\frac{d\Phi_B}{dt} = A \frac{dB}{dt} = (7.1\text{E-}4)(8) = 0.0057 \text{ Wb/s}$ . Here we use  $r_2$  for the area as it contains the flux within.

(c) (4 points). Calculate the electric field at point  $P_2$  at distance  $r_2$  from the center of the circles. What is its direction at that point? Draw it.

$$\text{We have } \oint \vec{E} \cdot d\vec{s} = E 2\pi r_2 = \frac{d\Phi_B}{dt}$$

$$E = 5.7\text{E-}3 / (2)(3.14)(0.015) = 0.06 \text{ N/C}$$

The direction is obtained similarly to the direction of the current in part a, so it would be pointing upward to the left.

### Possibly Useful Information

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$E = \frac{|q|}{4\pi\epsilon_0 r^2}, E = \sigma/\epsilon_0$$

$$\Delta U = U_f - U_i = -W$$

$$\Delta V = V_f - V_i = -W/q_0 = \Delta U/q_0$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$U_f + K_f = U_i + K_i$$

$$K = \frac{1}{2} mv^2$$

$$E = \frac{\Delta V}{\Delta s}$$

$$Q = CV$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_j} \text{ (series)}$$

$$u = \frac{1}{2} \epsilon_0 E^2$$

$$I = dQ/dt$$

$$\rho = \frac{1}{\sigma}$$

$$R = \frac{\rho L}{A}$$

$$P = IV$$

$$P_{emf} = I\mathcal{E}$$

$$\frac{1}{R_{eq}} = \sum \frac{1}{R_j} \text{ (parallel)}$$

$$I = (\mathcal{E}/R)e^{-t/RC}$$

$$I = (Q/RC)e^{-t/RC}, I_0 = (Q/RC)$$

$$E = \sigma/\epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12} (\text{C}^2 / \text{N} \cdot \text{m}^2)$$

$$\vec{E} = \vec{F}/q_0$$

$$\epsilon_0 \Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$U = -W_\infty$$

$$V = -W_\infty/q_0$$

$$V = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$U = -W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$C_{eq} = \sum C_j \text{ (parallel)}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

$$C = \kappa C_0$$

$$\Delta V = \mathcal{E} - Ir$$

$$V = IR$$

$$P = I^2 R = V^2/R$$

$$I = \frac{\mathcal{E}}{R + r}$$

$$R_{eq} = \sum R_j \text{ (series)}$$

$$q(t) = Q(1 - e^{-t/RC})$$

$$q(t) = Qe^{-t/RC}$$

$$\lambda = Q/L, \sigma = Q/A, \rho = Q/V$$



$$\vec{F}=q\vec{v}\times\vec{B}$$

$$|\vec{F}|=qvBsin(\theta)$$

$$r=mv/qB, \omega =qB/m$$

$$d\vec{F}=Id\vec{s}\times\vec{B}$$

$$\vec{\mu} = N I \vec{A}$$

$$\mathbf{B}=\frac{\mu_0I}{4\pi}\int\frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

$$B=\mu_0I/2\;\pi r$$

$$F/l=(\mu_0I_1I_2)/2\pi a$$

$$\varepsilon=\oint\vec{E}.d\vec{s}=-\frac{d\Phi_B}{dt}$$

$$\varepsilon=Blv$$

$$\vec{F}=\vec{IL}\times\vec{B}$$

$$\tau=\vec{\mu}\times\vec{B}$$

$$U=-\vec{\mu}\bullet\vec{B}$$

$$d\vec{B}=\mu_0\Big/4\pi\;\;Id\vec{s}\times\vec{r}\Big/\;r^3\;\;,\;\;\mu_0=\;\;4\pi\;\;\times\;\;10^{-7}$$

$$\mathrm{T.m/A}$$

$$B=\frac{\mu_0I}{4\pi a}(\cos\theta_1+\cos\theta_2)$$

$$B=\mu_0nI\;\;(\text{solenoid})$$

$$\oint\vec{B}.d\vec{s}=\;\;\mu_0I_{\rm enc}$$

$$\Phi_B=\int\vec{B}.d\vec{A}$$

$$\Phi_B=\oint\vec{B}.d\vec{A}=0$$