Problem 1. For the capacitor,

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$$

(a) $$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \frac{0.200 \text{ A}}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)\left[\pi(10.0\times10^{-2} \text{ m})\right]}$$

$$= \boxed{7.19\times10^{11} \text{ V/m}\cdot\text{s}}$$

(b) $$\oint B\cdot ds = \epsilon_0\mu_0\frac{d\Phi_E}{dt}: \qquad 2\pi rB = \cancel{\epsilon_0}\mu_0\frac{d}{dt}\left[\frac{Q}{\cancel{\epsilon_0}A}\cdot\pi r^2\right]$$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0(0.200 \text{ A})(5.00\times10^{-2} \text{ m})}{2\left[\pi(10.0\times10^{-2} \text{ m})^2\right]} = \boxed{2.00\times10^{-7} \text{ T}}$$

Problem 2. The net force on the proton is the Lorentz force, as described by

$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \quad \text{so that} \quad \vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right]$$

Taking the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$,

$$\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\hat{\mathbf{j}} + 200(0.300)\hat{\mathbf{k}}$$

$$\vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right] = \left(\frac{1.60\times10^{-19}}{1.67\times10^{-27}}\right)\left[50.0\,\hat{\mathbf{j}} - 80.0\,\hat{\mathbf{j}} + 60.0\,\hat{\mathbf{k}}\right] \text{ m/s}^2$$

Then, $$= \boxed{\left(-2.87\times10^9\hat{\mathbf{j}} + 5.75\times10^9\hat{\mathbf{k}}\right) \text{ m/s}^2}$$

Problem 3. (a) f = c/λ = 3E8/48 = 6.25MHz.

(b) B = E/c = 20/3E8 = 66.7 nT. E X B is in direction of propagation so -j X -k = i. Direction is in minus z.

(c) Need k = 2Pi/λ = 2 Pi/48 = 0.131 m. ω = 2Pif = 3.93E7. B is in the z direction.

B – 66.7 cos(0.131x-3.97E7t) **k**

Problem 4. (a) B = E/c = 116/3E8 = 3.87E-7 T

(b) λ = 2Pi/ = 6.28/1.4E7 = 4.49E-7 m

(c) f = c/λ = 3E8/4.49E-7 = 6.68E14

Problem 5.

(a) The desired quantities can be calculated from the dot product:

$$\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = (E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}) \cdot (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$$

$$= E_xB_x + E_yB_y + E_zB_z$$

Inserting values, we get:

$$\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = \left[(26.0\ \text{N/C})(0.160\ \mu\text{T}) + (-52.0\ \text{N/C})(0.580\ \mu\text{T}) + (65.0\ \text{N/C})(0.400\ \mu\text{T})\right] = 0$$

So we have:

$$E_xB_x = (26.0\ \text{N/C})(0.160\ \mu\text{T}) = 4.1600\ \mu\text{T} \cdot \text{N/C}$$

$$E_yB_y = (-52.0\ \text{N/C})(0.580\ \mu\text{T}) = -30.1600\ \mu\text{T} \cdot \text{N/C}$$

$$E_zB_z = (65.0\ \text{N/C})(0.400\ \mu\text{T}) = 26.0000\ \mu\text{T} \cdot \text{N/C}$$

Recall the definition of the dot product:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos(\theta)$$

where θ is the smaller angle between the two vectors. When $\cos(\theta) = 0$, the dot product is zero, and this occurs when $\theta = 90°$.

Since $\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = 0$, the two vectors are mutually perpendicular.

(b) Apply the definition of the Poynting vector:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

$$= \frac{1}{\mu_0}\left(E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}\right) \times \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

Taking the cross product:

$$\begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{bmatrix} = (E_yB_z - E_zB_y)\hat{\mathbf{i}} - (E_xB_z - E_zB_x)\hat{\mathbf{j}} + (E_xB_y - E_yB_x)\hat{\mathbf{k}}$$

So we have:

$$\begin{aligned} E_yB_z - E_zB_y &= (-52.0 \text{ N/C})(4.00 \times 10^{-7} \text{ T}) - (65.0 \text{ N/C})(5.80 \times 10^{-7} \text{ T}) \\ &= -5.85 \times 10^{-5} \text{ T} \cdot \text{N/C} \end{aligned}$$

$$\begin{aligned} E_xB_z - E_zB_x &= (26.0 \text{ N/C})(4.00 \times 10^{-7} \text{ T}) - (65.0 \text{ N/C})(1.60 \times 10^{-7} \text{ T}) \\ &= 0 \text{ T} \cdot \text{N/C} \end{aligned}$$

$$\begin{aligned} E_xB_y - E_yB_x &= (26.0 \text{ N/C})(5.80 \times 10^{-7} \text{ T}) - (-52.0 \text{ N/C})(1.60 \times 10^{-7} \text{ T}) \\ &= 2.34 \times 10^{-5} \text{ T} \cdot \text{N/C} \end{aligned}$$

Inserting values, we find:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\left[(E_yB_z - E_zB_y)\hat{\mathbf{i}} - (E_xB_z - E_zB_x)\hat{\mathbf{j}} + (E_xB_y - E_yB_x)\hat{\mathbf{k}}\right]$$

$$= \left(\frac{1}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}\right)\left[(-5.85 \times 10^{-5} \text{ T} \cdot \text{N/C})\hat{\mathbf{i}} - (0 \text{ T} \cdot \text{N/C})\hat{\mathbf{j}} + (2.34 \times 10^{-5} \text{ T} \cdot \text{N/C})\hat{\mathbf{k}}\right]$$

$$= \left((-46.6)\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + (18.6)\hat{\mathbf{k}}\right) \text{ W/m}^2$$

Problem 6.

(a) Apply the relationship $f\lambda = c$:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.00 \times 10^{19} \text{ Hz}} = 3.00 \times 10^{-11} \text{ m} = 30.0 \text{ pm}$$

(b) Apply the relationship $f\lambda = c$:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.50 \times 10^9 \text{ Hz}} = 4.62 \times 10^{-2} \text{ m} = 4.62 \text{ cm}$$

Problem 7. (a) $$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{1.7 \text{ m}} \boxed{\sim 10^8 \text{ Hz}} \boxed{\text{radio wave}}$$

(b) 1 000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick.

$$f = \frac{3.00 \times 10^8 \text{ m/s}}{6 \times 10^{-5} \text{ m}} \boxed{\sim 10^{13} \text{ Hz}} \boxed{\text{infrared}}$$