Exam III Solutions
114 2001

1. **Sources of Magnetic Fields**
   a. The law of Biot Sarvart tells us that the field will be zero at p. The law involves a term $d\vec{s} \times \hat{r}$, and these are perpendicular to each other so their cross product is zero.
   b. The point is so close to the wire that we can assume we have an infinite wire. Then the magnitude of the field is $\mu_0 I/2\pi r = (2 \times 10^{-7})(3)/1 \times 10^{-3} = 6 \times 10^{-4}$ T.
   c. From Ampère's law, the magnetic field at point a is given by $B_a = \mu_0 I_a / 2\pi r_a$, where I_a is the net current flowing through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m / A}\right)(1.00\ \mathrm{A})}{2\pi(1.00 \times 10^{-3}\ \mathrm{m})} = 200\ \mu\mathrm{T}$$ towards the top of the page.

Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current flowing through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1.00\ \mathrm{A} - 3.00\ \mathrm{A} = -2.00\ \mathrm{A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m / A})(2.00\ \mathrm{A})}{2\pi(3.00 \times 10^{-3}\ \mathrm{m})} = 133\ \mu\mathrm{T}$$ towards the bottom of the page.

2. **Faraday's Law**
   a. As the bar moves, a magnetic force (given by the lorentz force law) acts upward on positive charges and downward on negative charges. (1) This separates the charges so that there is an Electric field set up where qE = qvB. (2) The direction of the field goes from positive to negative charges so it is downward. (3) No net force is necessary. Once the field is set up, everything is in equilibrium. There is no dissipated energy and no current in the bar, so no force counteracting its motion.
   b. Yes, here a force would be necessary to keep it moving. Energy is dissipated in the resistor, so energy (work) must be put into the system to keep it going. There will be an upward current in the bar, so the magnetic field exerts a force on the bar = IℓB to the right. Thus there must be a counter force applied to the left to keep the bar moving.
   c. From part (a) we saw qE = qvB, so Eℓ = ε = vBℓ. Thus, $I = \frac{\varepsilon}{R} = \frac{B l v}{R}$ and (1) solving for v we get v = 1.00 m/s. (2) The equation, ε = vBℓ, can also be derived from Farday's law. Lenz's law tells us that the induced current will

increase flux out of the page (since the movement of the bar increase flux into the page). Thus (by the right hand rule) the current is counter clockwise. This is consistent with the direction of the force of B on charges due to v. (3) This is just , $\varepsilon = vB\ell = 3V$.

3. **Inductance**

a. (1) As the flux through the ring changes, and emf is induced in the ring. According the Lenz's law, the emf creates an induced current that opposes the change that caused it – so it wants to reduce the change in flux, creating amagnetic field that opposes the one going through it. Thus our situation is analogous to two magnets whose like poles are facing each other: they repel each other and the ring flies off. (2) When the switch is opened and the current dies, Lenz's law says that a current will flow in the ring, producing an induced field in the same direction of the dying one (since it wants to maintain the flux through it. Thus we have two magnets aligned with opposite poles and the ring does not fly off.

b. The time it takes to reach I/e depends on the characteristic time of the circuit: L/R. In fact L/R is equal to I/e. As L increases the time increases. Since $L = \mu_0 n^2 Al$, as n is doubled L increases by a factor of 4 and so does the time to reach I/e. Thus the time to reach I/2 increases.

c. (1) $\tau = L/R = 2.00 \times 10^{-3}$ s = 2.00 ms

(2) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00 \text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = 0.176$ A

(3) $I_{\max} = \dfrac{\mathcal{E}}{R}$ = 6.00/4.00= 1.5 A

(4) $0.800 = 1 - e^{-t/2.00 \text{ ms}} \rightarrow t = -(2.00 \text{ ms}) \ln(0.200) = 3.22$ ms

4. **Electromagnetic waves.**

a. (1) There is no phase difference, the electric and magnetic fields are in phase in an Electromagnetic Wave. (2) the Poynting vector describes the magnitude and direction of energy transfer per unit area per unit time of an electromagnetic wave. The time average Poynting vector is the intensity of the wave. (3) It is better if it is reflective, then the momentum transfer is twice as much as if it is absorptive.

b. It will oscillate at 2ω. If the wave goes as cosine, the Poynting vector goes as cosine squared which has twice as many cycles per second. Mathematically, we see $\cos(2x) = \frac{1}{2} + \frac{1}{2}\cos^2(x)$.

c (1) $B = E/c = 100/3.00 \times 10^8 = 3.33\ \ 10^{-7}$ T = 0.333 μT

(2) $\lambda = 2\pi/k = 2\pi/1.00 \times 10^7 = 0.628\ \mu$m = 628 nm

(3) $f = c/\lambda = 3.00 \times 10^8/6.28 \times 10^{-7} = 4.77 \times 10^{14}$

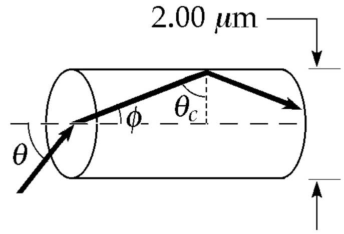
(4) $\omega = 2\pi f = 3.00\text{x} \times 10^{15}$ 1/s $\vec{B} = 0.333\ \mu\text{T} \sin(1.00 \times 10^7 x - 3.00\text{x} \times 10^{15} t)\hat{k}$

5. Nature of Light

a. (1) No, it only bends towards the normal when it goes into a medium with a higher index of refraction. It bends away from the normal if it enters a medium of lower index of refraction. (2) Its frequency does not change. If it did, cycles would accumulate at the boundary (or not have enough to get through). (3) The wavelength changes since the (4) speed changes.

b. (1) Rays 2, and 4 are due to reflection and 3, 5 and 6 are due to refraction. (2) They are essentially equal as they are drawn. If you measure them from the horizontal axis, counterclockwise, the angle θ_1 when measured clockwise is equal to θ_2 measured counterclockwise – that is $\theta_2 = 180^{o} - \theta_1$. (3) They are equal as drawn. The angle $\theta_3 = -\theta_2$. These angles, θ_2 and θ_3, would not change, (4) blue light would be be refracted more so that the displacements of the rays 5 and 6 would move closer to ray 1 but the angles stay the same.

c. (1) We have $\sin(\theta) = 1.36 \sin(\phi)$. So $\phi = \sin^{-1}(1/1.36*\sin(\theta))$.



(2) $\sin(\theta_c) = 1/1.36 = 0.735$ so $\theta_c = 47.3°$

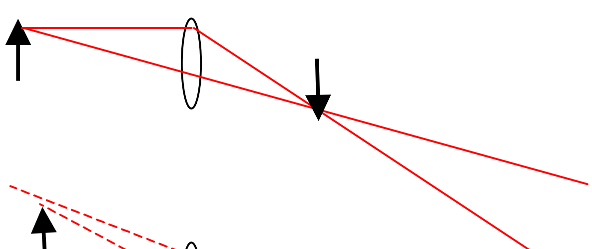
Geometry shows that the angle of refraction at the end is

$\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

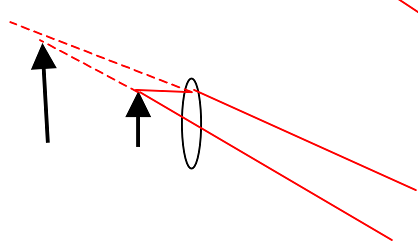
Then, Snell's law at the end, (part 1) gives $\theta = \sin^{-1}(1.36*\sin(\phi))$, so $\theta = 67.2^{o}$.

6. Geometrical Optics

a. (1)



(2)



b. $1/p + 1/q = 1/f$, so $1/q = 1/f - 1/p$. Here all of these are positive and stay positive since the image is real. (1) Since p changes, q changes. (2) since $m = -q/p$, m changes too (we need to verify in part three that they don't cancel each pother out). (3) As the object moves closer to the lens, p decreases and

1/p increases, so 1/q decreases and **q increases.** Thus m = -q/p so, with q increasing and p decreasing, the object gets bigger.

c. (1) $q = (p + 5.00\text{ m})$ and, since the image must be real, m = - q/p = -5.or q = 5p. Therefore, $p + 5.00 = 5p$ or $p = 1.25$ m and $q = 6.25$ m. From 1/p + 1/q = 1/f = 2/r we get R = 2pq/(q+p) = 2.08 m. It has to be concabe to form a real image. (2) From part (1), $p = 1.25$ m; the mirror should be 1.25 m in front of the object.