Public Choice **111:** 377–397, 2002.

Strategic competition in sequential election contests

KOLEMAN S. STRUMPF*
Department of Economics, University of North Carolina at Chapel Hill, NC 27599-3305, U.S.A.; e-mail: cigar@unc.edu

Accepted 3 February 2001

Abstract. This paper studies a sequential election contest, such as the American presidential primary, in which several elections occur one at a time until a single winner emerges. The conventional wisdom is such a system benefits a candidate favored in the initial elections because of momentum. This paper uncovers a potentially opposing force if participation is costly and candidates exit when they have unfavorable future prospects. A candidate with friendly elections at the end of the contest will typically benefit from the resulting game theoretic competition. Tension between this strategic effect and momentum helps explain several empirical regularities of presidential primaries.

1. Introduction

The most fundamental characteristic of an electoral system is how it translates voter preferences into election outcomes. As Myerson (1995) notes in his recent survey of democratic institutions, formal analysis of this question is one of the central pillars of social choice research. This paper contributes to the literature by exploring a sequential game in which several elections are contested one at a time until a single winner emerges. The motivation for this work is one such sequential contest, the U.S. presidential primary.

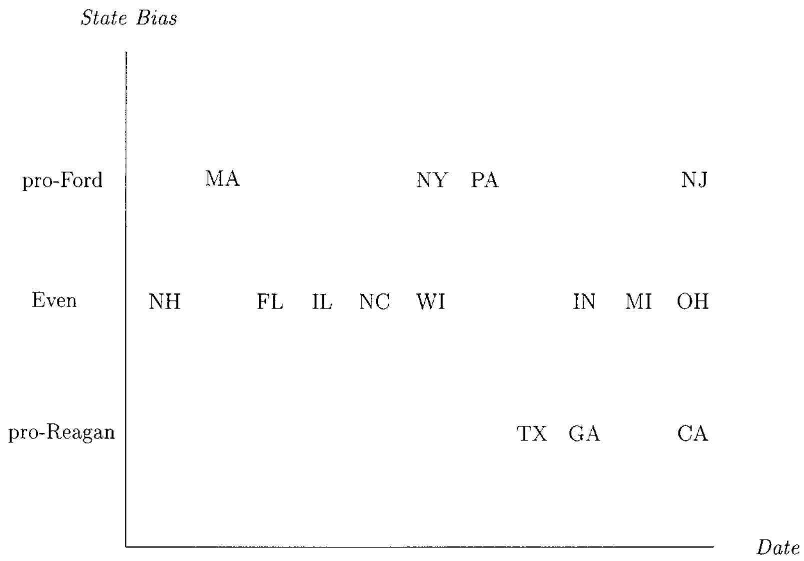
The two major American political parties select their nominee though a state-by-state contest. A peculiar feature of the primary system is that state elections are staggered in time, possibly allowing for inter- election spillovers. In the conventional story, early victories increase support in all subsequent states. This is because the winner in the first elections receives additional media attention and funding from campaign contributors, both

* I thank Gary Biglaiser, Steve Coate, James Friedman, Alexander Kovalenkov, Claudio Mezzetti, Thomas Piketty, James Snyder, participants in the 1995 Northwestern MEDS Analysis of Political Institutions and 1996 Public Choice conferences, as well as Harvard/MIT Political Economy and UNC Chapel Hill Theory seminars for comments and suggestions. Graduate fellowships from the National Science Foundation and the Harvard/MIT RTG in Positive Political Economy helped support my research. A previous version of this paper circulated under the title "Sequential Election Contests: Strategic Effects Oppose Momentum in Presidential Primaries."

of which increase the subsequent level of voter support.[1] These kinds of spillovers, known as *momentum*, are not the direct result of candidate behavior. The boost to the victor of early elections can potentially have important implications for the contest. Aldrich (1980) shows that certain sequences yield a nominee who would have lost in a simultaneous contest.

However, there are three reasons to believe other forces may be operating. First, there is a tenuous relationship between initial electoral success and eventual nomination success. For example the victor in the Iowa caucus and the New Hampshire primary, the first two elections, often does not gain the party nomination. In the 11 contested elections since 1972 (the first year in which state votes bound convention delegates), 5 losers in each of New Hampshire and Iowa eventually clinched the nomination. The 1976 Republican contest between Gerald Ford and Ronald Reagan further illustrates the empirical weakness of momentum. Before the primary began, Ford led both in campaign resources and national polling (see Aldrich, 1980, Figure 5.3 and Federal Election Commission, 1977). As Figure 1 shows, many of the early primary states favored Ford. The conventional wisdom predicts that Ford would win the initial elections, and the resulting momentum would propel him to an easy victory. While Ford did win the first four states, Reagan managed to avoid a bandwagon, garner key wins in North Carolina and Texas, and keep the contest competitive all the way to the last election in his home state of California.[2] A second argument for additional forces is the stability of the primary schedule. If early elections play a disproportionate role in the process, states should leapfrog to the beginning of the schedule in order to gain more influence. Figure 2 shows the primary schedules in 1972, 1996 and 2000. While some states did move to an earlier place between 1972 and 1996, many others maintained their place at the end of the schedule and none displaced New Hampshire as the first primary. The order of states remained relatively constant between 1996 and 2000. The third issue is the candidates' state of residence. If momentum is the central feature of primaries, then favorite-sons from early states like New Hampshire and Iowa would have a decided advantage over those from later states. In fact no recent nominee and few actual candidates have been from an early primary state.

One missing feature in the momentum story is a role for candidate decisions. While candidates benefit from winning the overall contest, participation in the race is costly in terms of time and money as well as political or personal capital. A candidate may choose to exit after weighing his future costs and benefits. If he is unlikely to win the contest while participation costs are high, than he should quit. This decision is influenced by the other candidates' actions. A candidate will have higher future costs and a lower probability of winning if one of his opponents can commit to participating



Sources: Aldrich (1980), Table 6.4; Bartels (1988), Table 9.3.
Note: Does not include small, late states.

Figure 1. The 1976 Republican Primary.

in all of the remaining elections. Because this behavior explicitly involves candidate interaction (unlike momentum), I will refer to it as the "strategic effect." The main result of this paper is that the strategic effect typically benefits a candidate who is the favorite in the final set of elections. Alternatively momentum is disadvantageous in such *back-loaded* sequences, since the candidate is likely to lose early elections where he is not the favorite. This tension between momentum and the strategic effect means that a sequential election system has an ambiguous effect on who will win the contest and serves as one explanation for the stylized facts highlighted in the last paragraph.

There are two main explanations for why the strategic effect benefits a candidate who participates in a back-loaded election sequence. The first rationale is the value of commitment. A candidate is unlikely to win the contest if several different contingencies lead him to exit the race, but he cannot credibly threaten to stay unless he has favorable future prospects. Only a candidate with friendly elections at the end can make such a pledge at every point in the contest. A second explanation for the strategic effect is the value of information. When a candidate receives bad news, such as losing an election in which he was favored, he may quit the race. Any early surprises will be

Date	*1972 Primary*	*1996 Primary*	*2000 Primary*
1 February			NH
8 February			DE
19 February			SC
20 February		NH	
22 February			AZ, MI
24 February		DE	
27 February		ND, SD, AZ	
29 February			VA, WA
2 March		SC	
5 March		CO, CT, GA, MA, ME, MD, RI, VT	
7 March	NH	NY	CA, CT, GA, MA, ME, MD MO, NY, OH, RI, VT
10 March			CO, UT
12 March		OR, FL, LO, MS, OK, TN, TX,	
14 March	FL		FL, LO, MS, OK, TN, TX
19 March		MI, OH, WI, IL	IL
21 March	IL		
26 March		CA, NV, WA	
4 April	WI		PA, WI
11 April	RI		
23 April		PA	
25 April	MA, PA		
2 May	OH, DC, IN		DC, IN, NC
4 May	TN		
6 May	NC		
7 May		DC, IN, NC	
9 May	NB, WV		NB, WV
14 May		NB, WV	
16 May	MD		OR
21 May		AR	
23 May	OR		AR, ID, KY
28 May		ID, KY	
4 June		AL, MT, NJ, NM	
6 June	CA, SD, NJ, NM		AL, MT, NJ, NM, SD

Sources: *Congressional Quarterly* 22 January 1972, and http://www.fec.gov.

Figure 2. 1972, 1996 and 2000 primary schedules.

favorable for a candidate facing a back-loaded sequence, since he is expected to lose these first elections.

I formalize this intuition in a war of attrition game from which momentum is purposely excluded. Two forward-looking candidates compete in a series of costly battles (the elections) for a single prize (the contest). Every election has a single winner with the ex ante probability of a win varying across elections. After the victor of each election is revealed, either candidate may choose to permanently quit. A candidate wins the contest if he wins a majority of the elections or his opponent quits. I focus on election sequences with two properties: (i) either candidate has an equal chance of winning if there are no quits; (ii) the elections are increasingly more favorable for candidate one. When costs are exogenous and fixed, I show that candidate one typically benefits from such back-loaded election sequences. I also find that the strategic advantage of back-loading typically holds in models where fighting costs are choice variables which influence election outcomes.

This paper is related to a recent literature on sequential elections. The main contrast is that the previous papers focus on voter behavior while my work considers candidate actions.[3] Bartels (1988) and Dekel and Piccione (2000) explore implications of sequential voting when voters have private information about the value of the alternatives. Because voters may strategically reveal their information, this is related to the literatures on herding (Banerjee, 1992; Bikhchandani et al. 1992) and sequential polling (Cukierman, 1991; McKelvey and Ordeshook, 1985a, 1995b).[4] These papers generally focus on the extent to which sequential elections reveal private information while I take no stand on the distribution or aggregation of information. But there are also more fundamental differences. For example, Dekel and Piccione (2000) show that the symmetric equilibria of simultaneous voting games are often equilibria of sequential contests. In contrast I show that the outcome in sequential and simultaneous elections can be different when there is strategic candidate behavior.

My modeling approach can be applied to other dynamic games with costly participation. For example, in which house of a bicameral government (such as the U.S. Congress) should a bill be initiated given their different ideological leanings? My theory suggests that the bill might pass with higher probability if it is first proposed in the less favorable venue. Another example would be to generalize war of attrition models to include non-stationarity. One prominent case is the patent race game where two firms engage in research competition (Fudenberg et al., 1983). If research progress is publicly observed, is it better to be a “quick” or “slow” researcher? This paper suggests that if a firm expecting quick results has initial difficulties, it will likely abandon the R&D process. The slower firm is willing to continue regardless of its initial success and may have an advantage in equilibrium.[5]

This paper is organized as follows. The next section presents a simple exogenous cost framework which illustrates the strategic advantage of back-loaded sequences. With this as motivation, Section 3 considers a more computationally involved model where costs are endogenously selected and may influence election outcomes. The strategic advantage of back-loaded sequences typically still holds in this more realistic framework. Section 4 reexamines the stylized facts of presidential primaries and discusses directions for future research.

2. Exogenous costs

2.1. *Setup*

Presume there are two candidates who benefit from clinching the unit valued contest but must pay a cost for participating in each of a sequence of elections.[6] Candidate i's preferences are, $U_i = I(i \text{ won}) -$ Accumulated Costs, where the first term is an indicator of whether i has won the contest. Each election, $t = 1, 2, \dots, T$, has exactly one winning candidate. These election outcomes are revealed sequentially in time with $W(t) \in [0, t-1]$ the cumulative win count of candidate one at the start of election t. The elections may be biased towards one of the candidates, with $p_t \in [0, 1]$ the probability of candidate one winning election t.[7] These probabilities are exogenous, commonly known and cannot be influenced by current actions or the history of play. An *upset* occurs when the less favored candidate wins an election. A sequence which is *back-loaded* for candidate one satisfies $p_t \leq p_{t+1}$. Participation costs, c, are constant for each election and common to both candidates.

The candidates' only decision is whether to remain in or quit the contest, and their choice may be contingent upon previous election outcomes. The quitting decision is irreversible. once out, a candidate cannot re-enter at a later time. There are two ways to win the contest. First, a candidate may win a majority of the elections; when this occurs, the leading candidate has a *mathematical clinch* while the loser suffers *mathematical elimination*. Second, if a candidate exits during election t, his opponent wins the contest and does not pay fighting costs in that or any later election.

The appropriate solution concept for this game is subgame perfection and the (unique) equilibrium may found by backwards induction.[8] A candidate will exit if and only if his expected future costs exceed his probability of winning the contest along the equilibrium path, and his opponent is committed to remain this election. That is candidate i's equilibrium choice is based on maximizing his continuation value, $V_{it}(W)$.[9]

2.2. *Three election example*

With this framework in place, I will analyze the equilibrium for the sequence $p_t = \{p, 0.5, 1-p\}$ where $p \in [0, 0.5)$. This sequence satisfies two properties:

1. **back-loaded for candidate one**: $p < 0.5 < 1 - p$
2. **unbiased on average**: when quits are not allowed, each candidate has an equal chance of winning the contest (I will refer to this as the *non-strategic* outcome).

The strategic effect in this back-loaded sequence benefits candidate one if his equilibrium probability of winning the contest exceeds one-half.

Costs are restricted to the interval, $c \in (p, 0.5(1+p)/(2-p))$. The lower bound, $c > p$, means that participation costs exceed the probability of an upset.[10] The upper bound is merely a technical formality which excludes some unreasonable cases where both candidates have negative continuation values.

The backwards induction solution begins with the final election. At $W(3) = 2$ [$W(3) = 0$] candidate one [two] has won the first two elections and so has mathematically clinched the contest. The eliminated candidate exits immediately, so his opponent incurs no additional fighting costs: $V_{13}(W = 2) = V_{23}(W = 0) \equiv 1$ and $V_{13}(W = 0) = V_{23}(W = 2) \equiv 0$. If instead the candidates split the first two elections, $W(3) = 1$, then neither has achieved a mathematical clinch. The candidates' continuation value when both contest this election are,

$$\widetilde{V}_{13}(W=1) \; = \; p\times 0 + (1-p)\times 1 - c \;\; > \;\; 0 \tag{1}$$

$$\widetilde{V}_{23}(W=1) \; = \; p\times 1 + (1-p)\times 0 - c \;\; < \;\; 0 \tag{2}$$

where $\widetilde{V}_{it}(W)$ is used to indicate continuation value when neither candidate is permitted to quit at the current stage. In these equations, the first [second] term is the payoff when candidate one [two] wins this last election. Since candidate two's costs exceed his probability of winning, he exits, $V_{23}(W = 1) = 0$, and candidate one wins the contest, $V_{13}(W = 1) = 1$.

These results can be used solve for optimal actions during the second election. If candidate two lost the first election, $W(2) = 1$, then he has no reason to remain now because he will surely exit from the final election. This means $V_{12}(W = 1) = 1$ and $V_{22}(W = 1) = 0$. If instead candidate two won the first election, $W(2) = 0$, then the loser of this election will immediately exit. Since $p_2 = 0.5$ both candidates have equal continuation values,

$$\begin{aligned}\widetilde{V}_{12}(W=0) \; &= \; 0.5\times V_{13}(W=0) + 0.5\times V_{13}(W=1) - c \\ &= \; 0.5 - c \; > 0 \end{aligned}\tag{3}$$

$$\widetilde{V}_{22}(W=0) \; = \; 0.5 - c \; > 0 \tag{4}$$

Because these terms are positive, both candidates contest the election and receive a continuation value $V_{i2}(W = 0) = 0.5 - c$.

Finally in the first election,

$$\begin{aligned}\widetilde{V}_{11}(W=0) &= (1-p)\times V_{12}(W=0) + p\times V_{12}(W=1) - c \\ &= 0.5(1+p) - (2-p)c \;> 0 \end{aligned} \tag{5}$$

$$\begin{aligned}\widetilde{V}_{21}(W=0) &= (1-p)\times V_{22}(W=0) + p\times V_{22}(W=1) - c \\ &= 0.5(1-p) - (2-p)c \;\gtrless 0 \end{aligned} \tag{6}$$

where the inequalities follow from the assumptions on c. There are two possibilities to consider. First, both candidates may enter the initial election in which case candidate one's probability of eventually winning the contest is $0.5(1+p) \geq 0.5$. Alternatively if costs are high enough, then (6) is negative and so candidate two will not enter the first election. Candidate one is the unopposed contest winner.

Result 1. In the sequence $p_t = \{p, 0.5, 1-p\}$ where $p \in [0, 0.5)$ and $c \in (p, 0.5(1+p)/(2-p))$, the equilibrium outcome is:

$$\Pr(1\text{ win}) = \begin{cases} 0.5(1+p) & c \leq \dfrac{0.5(1-p)}{2-p} \\ 1 & c > \dfrac{0.5(1-p)}{2-p} \end{cases} \tag{7}$$

Figure 3 plots the solution for a wider range of c values. Despite the rich array of possible equilibria, in most of the parameter space candidate one wins the contest with probability at least one-half, the non-strategic outcome.[11]

There are two explanations for this advantage to back-loading. The first rationale is the *value of commitment*. Even when competition is expensive, an early loss does not lead candidate one to exit since he still has two reasonably favorable elections; he can credibly pledge to fight the remainder of the sequence. Alternatively, if candidate two is upset in the beginning he faces difficult elections and an unyielding opponent. Thus two quits if he loses the first election or is tied coming into the last election. A related intuition is the *value of information*. A win for candidate two in the first election does not induce his opponent to quit since this was the expected outcome. An upset, however, is a large surprise and will discourage candidate two. It is this unexpected shift in relative positions which induces quits, and so it is advantageous to have any possible early shock be favorable. By definition back-loaded sequences satisfy this property, since the early elections are relatively unfavorable.

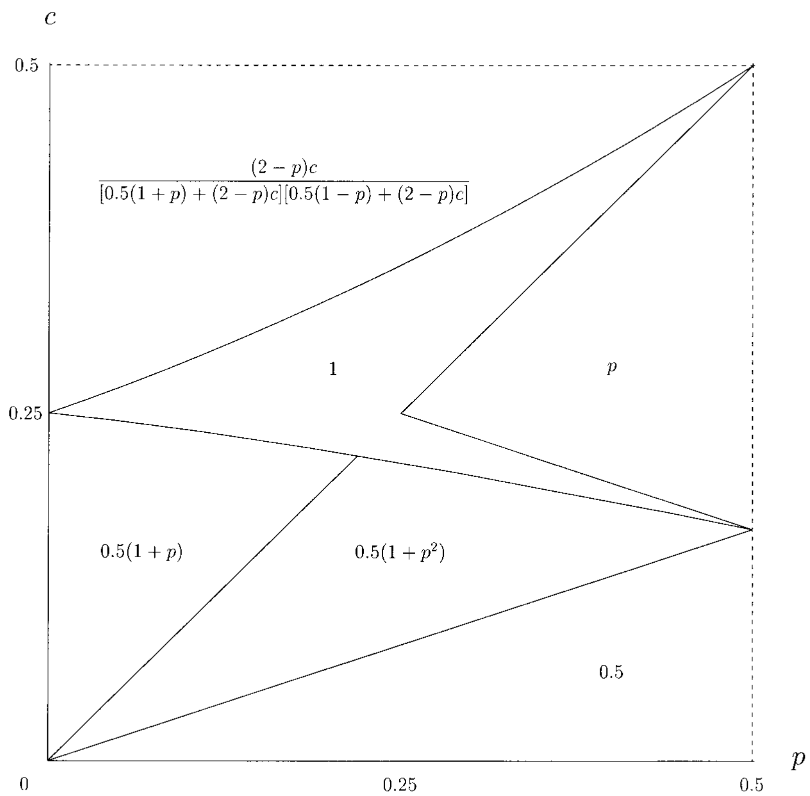


Figure 3. Pr(1 Win) under {p, 0.5, 1 − p} with exogenous costs.

2.3. *General horizon contests*

An interesting and important extension is to see if the strategic advantage of back-loading extends to arbitrary length sequences. Because of space constraints, such results cannot be reported here. An earlier version of the paper shows that back-loading is beneficial for two classes of T-length, unbiased on average, contests: (i) $p_t = \{p, \ldots, p, 0.5, 1-p, \ldots, 1-p\}$ where $p < 0.5$ so $0.5(T-1)$ elections favor each candidate; (ii) $p_t = t/(T+1)$. Full details of these results are available upon request.

3. Endogenous costs

A possible objection to the model in Section 2 is that participation costs are exogenous. I now weaken this assumption. Candidates will be allowed to select a spending level, contingent on the history of play, which influences the election outcome. Such expenses, however, are still costly. This endogenous cost structure adds to the model's realism. For example, it allows a candidate to employ a high cost strategy in an attempt to knock-out his opponent, a common practice of front-runners in presidential primaries (Wilcox, 1991). It also allows a more meaningful consideration of simultaneous election contests. More importantly, I show that the strategic advantage of back- loading still typically holds when costs are endogenous.

3.1. *Preliminaries*

The main modification of the model is allowing expenditures to influence election outcomes. At each election t candidate i must select a level of spending, $c_{it}(W) \geq 0$, which potentially depends on the cumulative win count W. Conditional on these spending decisions, candidate one's probability of winning election t is,

$$\Pr_t(W) = \frac{p_t c_{1t}(W)}{p_t c_{1t}(W) + (1-p_t)c_{2t}(W)} \equiv \frac{p_t}{p_t + (1-p_t)(c_{2t}(W)/c_{1t}(W))} \tag{8}$$

where $p_t \in [0, 1]$ is electorate t's predisposition to vote for candidate one.[12] There are several reasonable properties of (8):

Properties of $\Pr_t(W)$:

(i) A sufficient statistic for the influence of spending on vote outcomes is the ratio $c_{2t}(W)/c_{1t}(W)$. When candidates spend equally, the probability is simply based on voter preferences: $c_{1t}(W) = c_{2t}(W) \rightarrow \Pr_t(W) = p_t$;

(ii) The probability of winning increases in own spending and decreases in opponent spending: $\partial \Pr_t / \partial c_{1t} \geq 0$, $\partial \Pr_t / \partial c_{2t} \leq 0$;

(iii) There are decreasing returns to spending: $\partial^2 \Pr_t / \partial c_{1t}^2 \leq 0$, $\partial^2 \Pr_t / \partial c_{2t}^2 \geq 0$.

One important special case of this model is when $c_{1t}(W) = c_{2t}(W) \equiv c$ is imposed for every t and W. The game is then equivalent to the exogenous cost model studied in Section 2.

I will continue to presume candidates receive unit benefit from winning the contest and disutility from own expenditures. The continuation values at election t are,

$$V_{it}(W) = \left[\sum_{\Gamma \in \mathrm{Win}_{it}(W)} \prod_{u \in \Gamma_{1\,\mathrm{win}}} \Pr_u(W) \prod_{v \notin \Gamma_{1\,\mathrm{win}}} (1 - \Pr_v(W)) \right] - E \sum_{s=t}^{T} c_{is}(W) \tag{9}$$

where $\mathrm{Win}_{it}(W)$ is the set of future paths in which candidate i wins the contest and E is the expectation over all future paths when the strategy pair $\mathbf{c}_{.}$ is used.

One advantage of this setup is that it allows a meaningful consideration of the simultaneous contest in which all spending decisions are made at once and candidates maximize their probability of winning the contest. This static game has a remarkably simple equilibrium.[13]

Result 2. Assume that each candidate simultaneously chooses a vector of spending levels, $c_{i1}, \ldots, c_{iT}$, with the objective of maximizing (9) for $t = 1$. In equilibrium, the candidates will exactly match each others spending in every election, $c_{1t} = c_{2t}\ \forall t$.

Proof: This is a special case of Snyder (1989) Proposition 4.1.

Result 2 implies that either candidate's probability of winning the contest in the simultaneous spending game is the same as in the non-strategic case where there are no exits.[14]

A few preliminary points will ease the analysis. The inductive form of candidate i's continuation value in (9) is,

$$V_{it}(W) = Pr_t^*(W)V_{it+1}(W+1) + (1 - Pr_t^*(W))V_{it+1}(W) - c_{it}^*(W) \quad (10)$$

where * indicates an equilibrium value and all terms implicitly depend on the strategy of the other candidate. The first-order conditions for optimal spending are,

$$\begin{aligned} \frac{\partial V_{1t}(W)}{\partial c_{1t}(W)} &: \quad \triangle V_{1t+1}(W)\frac{\partial\, Pr_t(W)}{\partial c_{1t}(W)} - 1 = 0 \\ \frac{\partial V_{2t}(W)}{\partial c_{2t}(W)} &: \quad \triangle V_{2t+1}(W)\frac{\partial (1 - Pr_t(W))}{\partial c_{2t}(W)} - 1 = 0 \end{aligned} \quad (11)$$

where $\triangle V_{1t+1}(W) \equiv V_{1t+1}(W+1) - V_{1t+1}(W)$ and $\triangle V_{2t+1}(W) \equiv V_{2t+1}(W) - V_{2t+1}(W+1)$ are the differential benefit of winning another election. (11) has a simple interpretation: candidates should set spending so that the marginal benefit– the value and increased probability of winning this election (the first term)– equals the marginal cost (the second term). Each reaction function has a unique solution since the probability term is concave in own spending and $\triangle V_{it+1}(W) \geq 0$ with equality only when one candidate has been mathematically eliminated (a formal proof of this result follows from mathematical induction and is available upon request).

Now the marginal returns to spending are,

$$\begin{aligned}\frac{\partial \mathrm{Pr}_t}{\partial c_{1t}} &= \frac{p_t(1-p_t)c_{2t}}{(p_t c_{1t} + (1-p_t)c_{2t})^2} \\ \frac{\partial(1-\mathrm{Pr}_t)}{\partial c_{2t}} &= \frac{p_t(1-p_t)c_{1t}}{(p_t c_{1t} + (1-p_t)c_{2t})^2}\end{aligned} \tag{12}$$

Substituting (12) into (11) and solving yields explicit formulae for optimal expenditure,

$$c^*_{it}(W) = \left[\frac{\triangle V_{it+1}(W)}{p_t \triangle V_{1t+1}(W) + (1-p_t)\triangle V_{2t+1}(W)}\right]^2 p_t(1-p_t)\triangle V_{-it+1}(W) \tag{13}$$

and continuation values,

$$V_{1t}(W) = V_{1t+1}(W) + \frac{p_t^2 \triangle V_{1t+1}(W)^3}{[p_t \triangle V_{1t+1}(W) + (1-p_t)\triangle V_{2t+1}(W)]^2} \tag{14}$$

$$V_{2t}(W) = V_{2t+1}(W+1) + \frac{(1-p_t)^2 \triangle V_{2t+1}(W)^3}{[p_t \triangle V_{1t+1}(W) + (1-p_t)\triangle V_{2t+1}(W)]^2} \tag{15}$$

Each candidate's expenditure is increasing in his own differential benefit of winning but ambiguous in his opponent's differential benefit. prospects are also increasing in his own marginal benefit a win but decreasing in his opponent's win differential. One implication of the equations will be crucial for the intuition. From (13) relative spending is,

$$\frac{c^*_{2t}(W)}{c^*_{1t}(W)} = \frac{\triangle V_{2t+1}(W)}{\triangle V_{1t+1}(W)} \tag{16}$$

Prior to a mathematical clinch, each $\triangle V_{it+1}(W) > 0$ and the right hand side of (16) is positive and finite. This means neither candidate will quit (spend zero). Substituting (16) into (8) yields candidate one's equilibrium probability of winning election t,

$$\mathrm{Pr}^*_t(W) = \frac{p_t}{p_t + (1-p_t)(\triangle V_{2t+1}(W)/\triangle V_{1t+1}(W))} \tag{17}$$

3.2. *Simple three election back-loading*

I will iteratively apply (14), (15) and (17) to find the equilibrium for the back-loaded sequence from Section 2.2: $p_t = \{p, 0.5, 1 - p\}$ where $p \in [0, 0.5)$. Recall that under this sequence both candidates have an equal chance of winning the contest when there is no strategic behavior or there are simultaneous elections with endogenous costs.

The backwards induction solution under endogenous costs is summarized in Table 1. Candidate one wins the contest when he has at least two election victories. This occurs with probability,

$$\Pr^*(1\text{ win}) = \Pr_1^*(0)\Pr_2^*(1) + \Pr_1^*(0)[1 - \Pr_2^*(1)]\Pr_3^*(1) + [1 - \Pr_1^*(0)]\Pr_2^*(0)\Pr_3^*(1)$$

Substitution from the last column of Table 1 and some algebra yields the following result.

Result 3. When there are endogenous costs, the equilibrium for the sequence $\{p, 0.5, 1 - p\}$ where $p \in [0, 0.5)$ favors candidate one,

$$\Pr^*(1\text{ win}) = \frac{1 + 5p + 9p^2 - 40p^3 + 30p^4 + 6p^5 - 14p^6 + 4p^7}{2(1 + 4p + 6p^2 - 20p^3 + 10p^4)} \geq 0.5 \tag{18}$$

The inequality is strict unless $p = 0$.

Some intuition for this result follows from the equilibrium spending incentives listed in (16). Candidates who benefit more from a win, in terms of increased continuation value, spend more and due to (17) have a greater chance of winning the election. If no candidate has mathematically clinched by the last election ($W(3) = 1$, row two of Table 1), the victor wins the contest and so both are willing to spend the same amount. This means the equilibrium win probability exactly matches the electorate's preferences, $\Pr_3^*(1) = 1 - p \equiv p_3$. The situation is more interesting in the second election due to the asymmetry of future prospects. When candidate one has won the first election ($W(2) = 1$, row four of Table 1), he has an almost insurmountable lead since just one more win gives him a mathematical clinch and he is favored in the final election. Candidate two has a relatively small benefit from a current win since he would still be an underdog in the final election. Candidate one's differential benefit of a victory is significantly larger, and so he greatly out-spends his opponent;[15] this is the leader knock-out strategy mentioned earlier in the section. The implication is that candidate one's probability of victory greatly exceeds that based simply on electorate tastes, $\Pr_2^*(1) \gg 0.5 \equiv p_2$.

Table 1. Solution to endogenous cost game: $p_t = \{p, 0.5, 1 - p\}$ where $p \in [0, 0.5)$. Continuation values for the first period omitted.

		Net Benefit of Winning Current Election			Continuation Value		Probability 1 Wins Election
t	W	$\triangle V_{1t+1}(W)$		$\triangle V_{2t+1}(W)$	$V_{1t}(W)$	$V_{2t}(W)$	$\Pr_t^*(W)$
3	2	0	$=$	0	1	0	1
3	1	1	$=$	1	$(1-p)^2$	p^2	$1-p$
3	0	0	$=$	0	0	1	0
2	1	$p(2-p)$	$\geq$	p^2	$\frac{4-8p^2+6p^3-p^4}{4}$	$\frac{p^4}{4}$	$\frac{2-p}{2}$
2	0	$(1-p)^2$	$\leq$	$1-p^2$	$\frac{(1-p)^4}{4}$	$\frac{1+2p+4p^2-2p^3-p^4}{4}$	$\frac{1-p}{2}$
1	0	$\frac{3+4p-14p^2+10p^3-2p^4}{4}$	$>$	$\frac{1+2p+4p^2-2p^3-2p^4}{4}$	—	—	$\frac{3p+4p^2-14p^3+10p^4-2p^5}{1+4p+6p^2-20p^3+10p^4}$

Alternatively when candidate two has won the first election ($W(2) = 0$, row five of Table 1), he has the upper hand and spends more.[16] But his advantage, both in terms of continuation value and probability of winning, is not as significant as one's was when $W(2) = 1$. The reason is that candidate one is favored in the last election, so his differential benefit of winning when $W(2) = 0$ is nearly as large as his opponent's.[17] The spending ratio is much closer to one here than when $W(2) = 1$ (see notes 15 and 16), so candidate two receives a relatively small boost in election probability. In addition, the absolute level of spending is larger when $W(2) = 0$ than when $W(2) = 1$ because the candidates have such similar future prospects. So both candidates have small continuation values when $W(2) = 0$ while candidate one has a value near unity (and two near zero) when $W(2) = 1$.

The final step is to consider the first election (row six of Table 1). Because the return to winning is much larger for candidate one, he is willing to spend more. As a result, one's winning probability in this election exceeds the electorate bias, $Pr_1^*(0) \geq p \equiv p_1$ (with equality only at $p = 0$). It is largely candidate one's advantage in this first election which increases his equilibrium probability of winning the contest.[18]

3.3. *More general three election back- loading*

The tedious calculations in the last sub-section suggest that it will be infeasible to prove analytical results for endogenous cost contests with an arbitrary number of elections (an Appendix, which is available upon request, discusses numerical results which illustrate the strategic advantage of back-loading for the two general horizon sequences discussed in Section 2.3). Instead I will focus on the most general class of back-loaded, unbiased on average, three election contests: any $\{p_1, p_2, p_3\}$ which satisfies,

1. **back-loaded for candidate one**: $p_1 \leq p_2 \leq p_3$
2. **unbiased on average**: $p_1p_2p_3 + p_1(1 - p_2)p_3 + p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 = 0.5$

This class has the convenient property that the direction of the strategic effect is determined simply by whether candidate one's equilibrium probability of winning the contest is greater or less than one-half. Notice that the sequence in the last sub-section is a special case of this class.

The equilibrium can again be uncovered by repeatedly applying (14), (15) and (17). While analytical solutions are possible, they are quite cumbersome and rather non-intuitive looking (these formulae are available upon request).[19] It is easier to consider the graphical summary of the equilibria presented in Figure 4. The contours in the figure represent candidate one's probability of winning the contest at all feasible combinations of p_1 and p_2 while the white

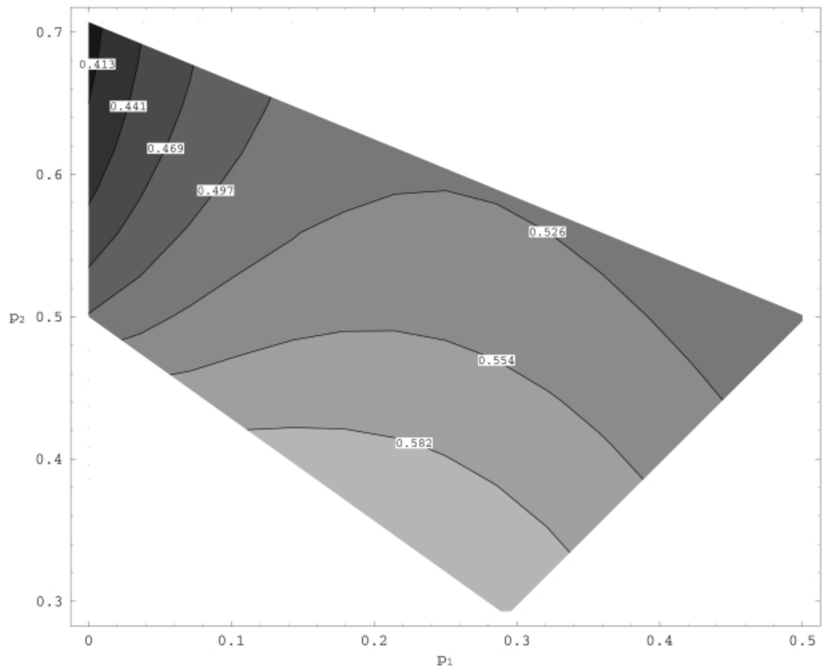


Figure 4. Pr(1 Win) under unbiased on average, back-loaded, three election sequences with endogenous costs.

regions represent parameter values which violate one or both of the above conditions.[20]

Two conclusions are apparent. First, candidate one's equilibrium win probability exceeds one-half for most of the parameter space. That is, most back-loaded sequences impart a strategic advantage. The intuition follows the same backwards induction reasoning from the last sub-section. The strategic effect is the result of the leading candidate spending more. While there is no differential spending in the final election, in the second election candidate one spends relatively more and enjoys a greater advantage when he is the leader then when two is the leader. This typically induces candidate one to spend relatively more in the first election. This combination of higher spending usually provides candidate one with a strategic advantage. The upper-left parameter space is the only exception to this reasoning. In this region p_1 is near zero which implies that candidate two almost surely wins the first election. But this means the only relevant second period history is where candidate two is the leader, and then he out- spends one. In equilibrium, this gives candidate two an advantage. Notice that this exceptional region requires $p_2 > 0.5$ and is larger when p_2 increases. These results may be summarized:

Proposition 1. Consider any three period sequence which is back-loaded for candidate one and unbiased on average. When there are endogenous costs, there exists a unique $p_1^*(p_2)$ so that in equilibrium,

$$\Pr(1 \text{ win}) > 0.5 \quad \Longleftrightarrow \quad p_2 \leq 0.5 \ \text{OR} \ p_2 > 0.5, p_1 \geq p_1^*(p_2) \tag{19}$$

where $\partial p_1^* / \partial p_2 > 0$.
The proof is omitted because of length and is available upon request.

The second result from Figure 4 is that candidate one's equilibrium win probability is maximal when p_1 is moderately valued (conditional on some p_2 value). That is, when candidate one is either an extreme or very slight underdog in the first election he does relatively worse overall. The intuition for the first case was discussed in the last paragraph. Alternatively, as p_1 increases then p_3 must decrease because of the unbiased condition. The latter effect diminishes candidate one's continuation value in the second election, and thus reduces his equilibrium win probability.

4. Conclusion

This paper studies sequential election contests such as presidential primaries. The conventional wisdom is that time- staggering benefits the leader in early elections through momentum. This paper highlights a new effect stemming from strategic behavior under costly competition. Forward-looking candidates drop-out if future participation costs exceed the probability of clinching the contest. When a candidate has all of his best elections at the start of the sequence, an early upset forces him to exit since he has bleak future prospects. At the same time, a contender with favorable elections at the end can credibly commit to remain even after a string of early losses. This strategic effect typically benefits a candidate facing a back-loaded sequence. I show this reasoning applies whether participation costs are exogenous or rather they influence election outcomes and are endogenously chosen.

The models in this paper purposefully exclude momentum. Because the strategic effect and momentum tend to work in the opposite direction, it is not clear whether on net back-loaded election sequence are beneficial or unfavorable. This tension may help explain the stylized facts of presidential primaries presented in the introduction– the stability of the election schedule, the scarcity of nominees from early states, and the tenuous relationship between initial electoral success and eventual nomination success. Momentum alone is inconsistent with these facts because it suggests that the first few elections play a disproportionate role in determining the nominee. The strategic effect helps maintain a stable primary schedule because it can provide incentives for states to schedule late elections and can limit the importance of the first few states (and the likely success of one of their favorite sons). Because both momentum and the strategic effect can influence a presidential primary, an interesting empirical project would be to gauge their relative importance in practice.

There are several extensions which would make the strategic model more realistic. First, an initial period could be added to the endogenous cost model during which the candidates commit to a minimum spending in each election. The game would then proceed as before and higher spending could occur when a particular election is actually contested. A candidate who precommits to high spending may be able to convince his opponent to not enter the contest at all.[21] While this is likely to modify the equilibrium, Result 2 suggests that if all spending is pre-committed then there is no net effect. Second, it would be interesting to explore whether sequential elections tend to favor candidates taking extreme positions. In the context of the model, such a candidate might be a heavy favorite in a few elections and an underdog in the remainder. Heckelman (2000) shows in a related model of U.S. Senate contests that having a primary stage tends to promote selection of extreme candidates. Third, many primaries have more than two candidates seeking the nomination. In such campaigns the early elections serve primarily to "winnow out" a plethora of weaker candidates. Full treatment of the N-candidate game will likely require use of cooperative theory and sorting through multiple equilibria (see Cooper and Munger (2000) for simulation evidence of such multiplicity). Fourth, the outcomes of early elections allow candidates to refine their beliefs about the probability of winning the remaining elections. Such learning should result in a strategic variant of momentum as early losses lead a candidate to reassess downwards his future prospects, possibly inducing him to exit. Investigating these generalizations are interesting topics for future research.

Notes

1. The underlying presumption is that voters are imperfectly informed about the candidates. Because voters prefer a candidate they are familiar with, they will tend to support the front-runner.
2. The limited strength of momentum was also evident in the 1996 and 2000 Republican primaries. In 2000 John McCain was not able to capitalize on his early wins in New Hampshire and Michigan. Similarly, in 1996 challengers Steve Forbes and Pat Buchanan benefited from early states which were receptive to their message (Arizona and New

Hampshire respectively), but they could not generate any momentum and eventually dropped out of the contest.

3. The only theoretical paper I am aware of which explicitly models candidate strategies in sequential election contests is Brams and Davis (1982). In the context of a model which imposes symmetric candidate resources, they prove the existence of an equilibrium where there is equal spending in every election. In contrast my work focuses on the role of asymmetry in such contests. Aldrich (1980) examines a discrete choice model somewhat similar to my exogenous cost game, but he does not formalize the dynamics.
4. Note that these mechanisms provide one explanation for the momentum effect discussed earlier.
5. There are many other possible applications. When seeking a new national contract, the United Automobile Workers always begins negotiating with just one of the Big Three carmakers; the choice of starting with a weak or strong firm is similar to the bill introduction problem in the text. A firm's internal labor market is often modeled as a tournament competition in which the prize is the CEO position (Rosen, 1986); whether this system favors individuals skilled in lower or higher level tasks is analogous to the R&D problem.
6. This disallows running for symbolic value or to shape the party platform. In practice candidates with such objectives tend to be marginal contenders.
7. These biases stem from electorate policy preferences and candidate attributes such as charisma or campaigning skill. The reason election outcomes are uncertain – p_t can be interior to the unit interval – is that voters are imperfectly informed and candidates have access only to imperfect polls.
8. A subgame perfect equilibrium is a strategy pair from which neither candidate will deviate, even at histories reached with probability zero along the equilibrium path.
9. The index on the win count will be suppressed when it is an argument in the continuation value.
10. One justification for the cost lower bound is that it is the equilibrium outcome of an endogenous cost game discussed in an earlier version of this paper which is available upon request.
11. There are two regions of Figure 3 which violate this statement. The first exceptional region, in the upper-left portion where $c > 0.5(1 + p)/(2 - p)$, makes both (5) and (6) negative. This means mixed strategies are used in the first election, both candidates have an equal probability of winning the contest, and with some chance neither enters at all. Because there are entrants in all real- world elections, these parameter values are implausible. The second exceptional region, in the middle-right of the parameter space, is where $(1 - p)/3 < c < p$. In this case the loser of the first election quits. Candidate one has a low probability of winning ($p < 0.5$), since this election is biased against him.
12. All of the results of this section carry over to the more general voting function,

$$\Pr_t(W) = \frac{p_t c_{1t}(W)^{\gamma}}{p_t c_{1t}(W)^{\gamma} + (1 - p_t) c_{2t}(W)^{\gamma}} \qquad \gamma \in [0, 1]$$

where γ parameterizes the "effectiveness" of spending for both candidates. One interesting property of this form is that when $\gamma \to 0$ candidate spending has a negligible influence on election outcomes, a distinct possibility in reality (see Levitt, 1994). I restrict my attention to $\gamma = 1$ in the text to simplify the algebra.

13. The same equilibrium also holds if the candidates simultaneously set spending with the goal of maximizing the *expected number* of elections they win.
14. The intuition for Result 2 is that the optimal level of spending in each election will always equate marginal costs and benefits. Winning an election is beneficial only if it is "pivotal,"

i.e. its outcome determines the winner of the contest (conditioning on the other elections). Thus the marginal benefit of spending in election t is the probability the election is pivotal times the marginal return to spending. Because an election's pivotal status is symmetric between candidates and the marginal cost of spending is one, there will be equal spending in each election.

15. Calculations based on (13) show,

$$c^*_{12}(1) = p^2(2-p)^2/4 \geq p^3(2-p)/4 = c^*_{22}(1)$$

with equality only at $p = 0$. When $p > 0$, relative spending is $c^*_{12}(1)/c^*_{22}(1) = (2-p)/p \geq 1$.

16. Here,

$$c^*_{12}(0) = (1-p)^2(1-p^2)/4 \leq (1-p)^2(1+p)^2/4 = c^*_{22}(0)$$

which gives relative spending $c^*_{12}(0)/c^*_{22}(0) = 1/(1+p) \geq 1$ with equality only at $p = 0$.

17. If two wins the election he clinches the contest while if one wins he is the favorite in the final (and pivotal) election.
18. Candidate one's significant advantage at $W(2) = 1$ has only a small effect on his equilibrium win probability because he is likely to lose the first election due to the unfavorable electorate bias.
19. When written as a function of (p_1, p_2), candidate one's equilibrium probability of winning the contest is a ratio of two 15th order polynomials.
20. p_3 is implicitly determined by the unbiased on average condition.
21. This can also be interpreted as endogeneizing the candidate entry decision. Besley and Coate (1997) evaluate such a model.

References

Aldrich, J.H. (1979). A model of the U.S. presidential primary campaign. In S.J. Brams, A. Schotter and G. Schwodiauer (Eds.), *Applied game theory*. Physica-Verlag, Würzburg.

Aldrich, J.H. (1980). A dynamic model of presidential nomination campaigns. *American Political Science Review* 74: 651–669.

Aldrich, J.H. (1980). *Before the convention: Strategies and choices in presidential nomination campaigns*. The University of Chicago Press, Chicago.

Banerjee, A. (1992). A simple model of herd behavior. *Quarterly Journal of Economics* 107: 797–817.

Bartels, L.M. (1988). *Presidential primaries and the dynamics of public choice*. Princeton University Press, Princeton.

Besley, T. and Coate, S. (1997). An economic model of representative democracy. *Quarterly Journal of Economics* 112: 85–114.

Bikhchandani, S., Hirshleifer, D. and Welch, I. (1992). A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy* 100: 992–1026.

Brams, S.J. and Davis, M.D. (1982). Optimal resource allocation in presidential primaries. *Mathematical Social Sciences* 3: 373–388.

Cooper, A. and Munger, M. (2000). The (un)predictability of primaries with many candidates: Simulation evidence. *Public Choice* 103: 337–355.

Cukierman, A. (1991). Asymmetric information and the electoral momentum of public opinion polls. *Public Choice* 70: 181–213.

Dekel, E. and Piccione, M. (2000). Sequential voting procedures in symmetric binary elections. *Journal of Political Economy* 108: 34–55.

Federal Election Commission. (1977). Washington, DC. *1976 presidential campaign receipts and expenditures*, no. 7., edition, 1977 FEC Disclosure Series.

Fudenberg, D., Gilbert, R., Stiglitz, J. and Tirole, J. (1983). Preemption, leapfrogging, and competition in patent races. *European Economic Review* 22: 3–31.

Heckelman, J. (2000). Sequential elections and overlapping terms: Voting for US senate. *Journal of Economic Behavior & Organization* 42: 97–108.

Levitt, S.D. (1994). Using repeat challengers to estimate the effect of campaign spending on election outcomes in the U.S. House. *Journal of Political Economy* 102: 777–798.

McKelvey, R.D. and Ordeshook, P.C. (1985a). Sequential elections with limited information. *American Journal of Political Science* 29: 480–512.

McKelvey, R.D. and Ordeshook, P.C. (1985b). Elections with limited information: A fufilled expecations model using contemporaneous poll and endorsement data as information sources. *Journal of Economic Theory* 36: 55–85.

Myerson, R.B. (1995). Analysis of democratic institutions: Structure, conduct and performance. *Journal of Economic Perspectives* 9: 77–89.

Rosen, S. (1986). Prizes and incentives in elimination tournaments. *The American Economic Review* 76: 701–715.

Snyder, J.M. (1989). Election goals and the allocation of campaign resources. *Econometrica* 57: 637–660.

Wilcox, C. (1991). Financing the 1988 prenomination campaigns. In E.H. Buell Jr. and L. Sigelman (Eds.), *Nominating the president*. Knoxville: The University of Tennessee Press.