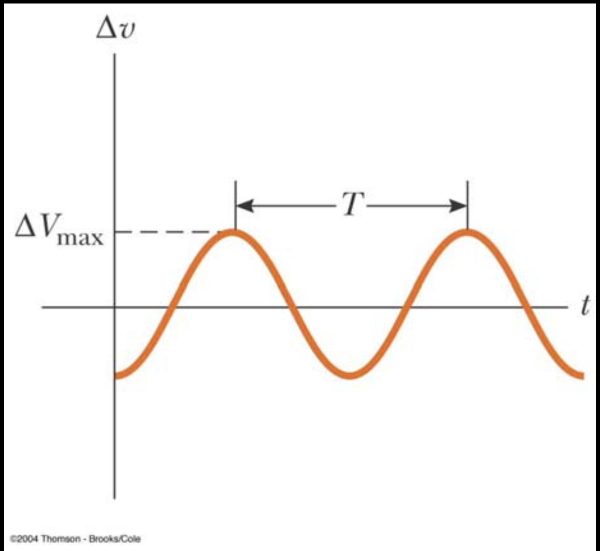
Alternating Current (AC) Circuits

- AC Voltages and Phasors
- Resistors, Inductors and Capacitors in AC Circuits
- RLC Circuits
- Power and Resonance
- Transformers

AC Sources

- AC sources have voltages and currents that vary sinusoidally.
- The current in a circuit driven by an AC source also oscillates at the same frequency.
- The standard AC frequency in the US is 60 Hz (337 rad/s).

$$\Delta v(t) = \Delta V_{max} \sin(\omega t)$$

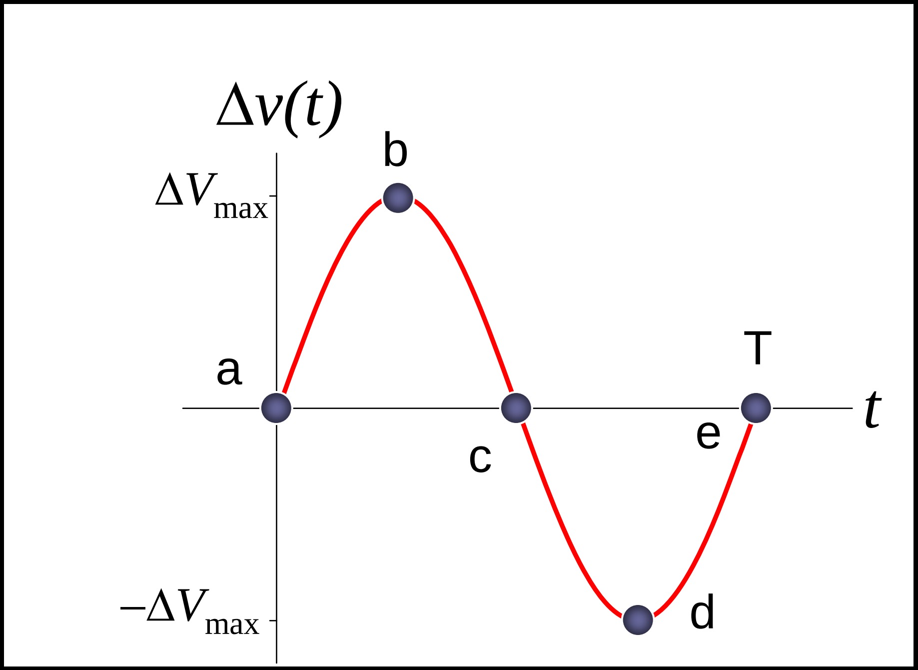


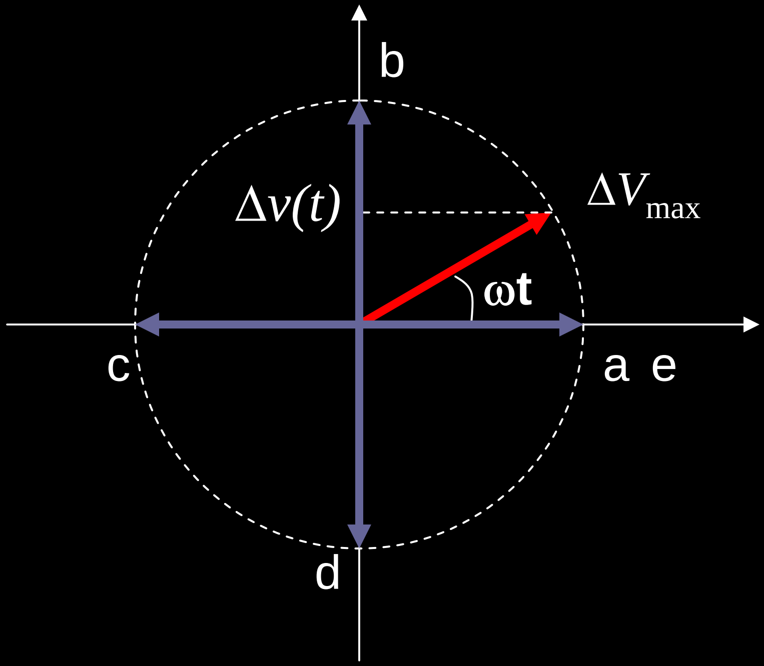
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Phasors

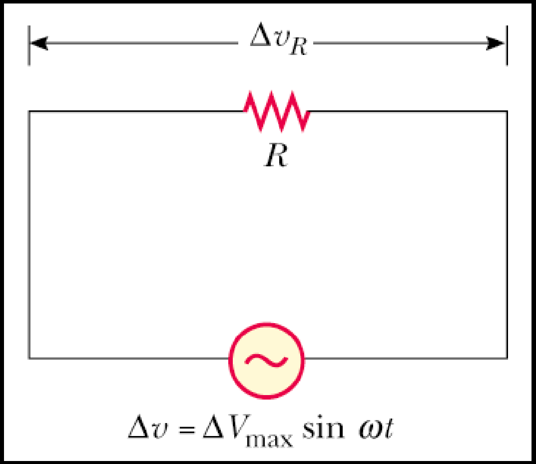
Consider a sinusoidal voltage source: $\Delta v(t) = \Delta V_{max} \sin(\omega t)$

This source can be represented graphically as a vector called a **phasor**:





Resistors in AC Circuits



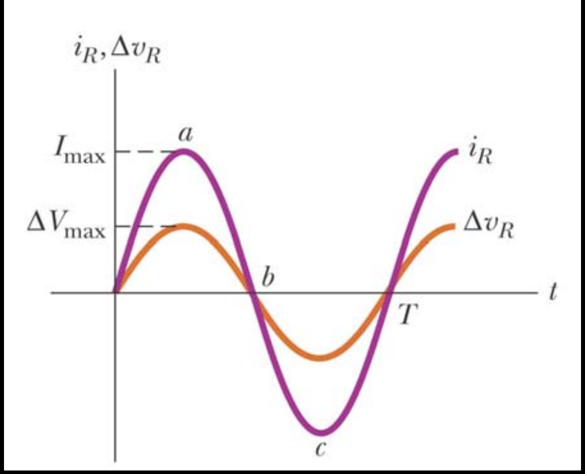
$$\Delta v - \Delta v_R = 0$$

$$\Delta V_{max} \sin \omega t = \Delta v_R$$

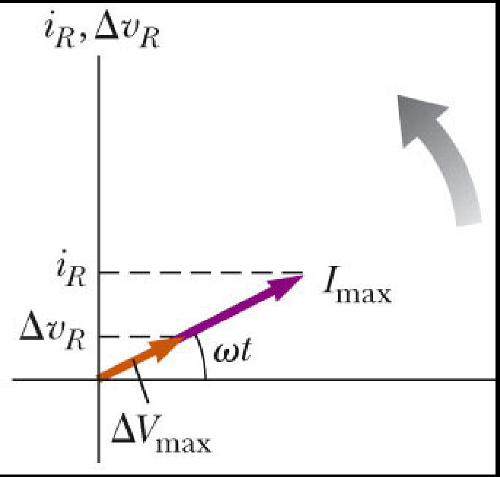
$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

$$I_{max} = \frac{\Delta V_{max}}{R}$$

$$\Delta v_R = I_{max} R \sin \omega t$$



Since both have the same arguments in the sine term, i_R and Δv_R are in line, they are **in phase**.

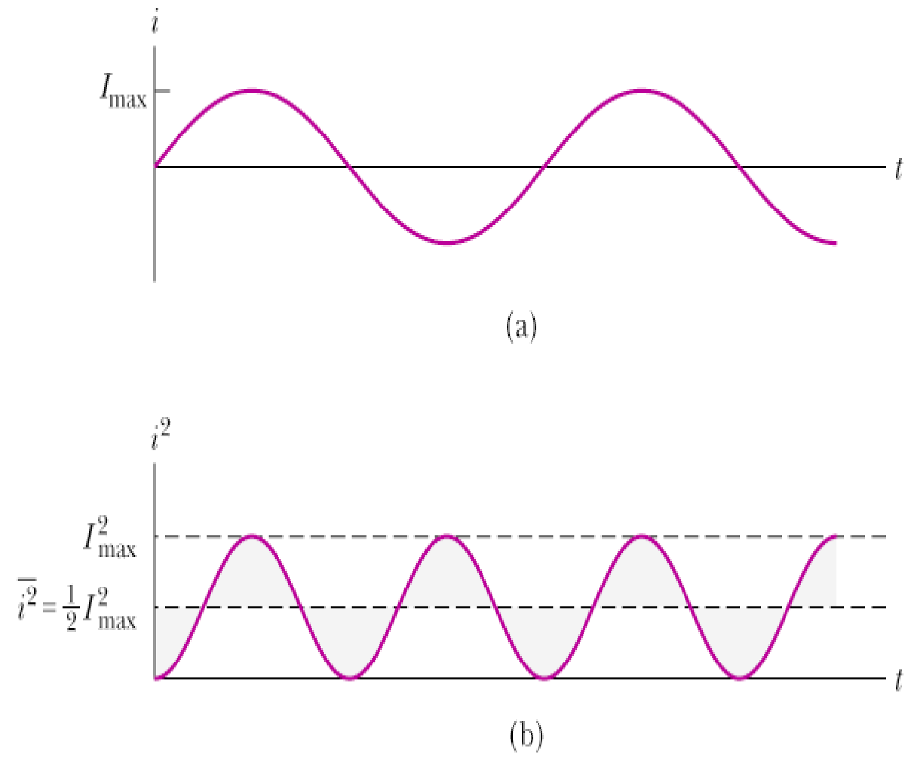


Root Mean Square (RMS) Current and Voltage

$$i_R = I_{\max} \sin \omega t$$

$$\mathcal{P}(t) = R{i_R}^2 = R{I_{\max}}^2 \sin^2 \omega t$$

$$\mathcal{P}(t) = R{i_R}^2 = R{I_{\max}}^2 \sin^2 \omega t$$

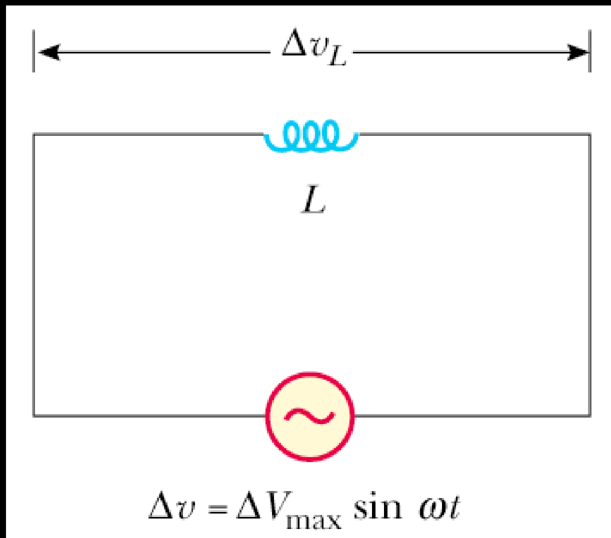


$$\mathcal{P}_{av} = R{I_{\max}}^2 \left(\frac{1}{2}\right) = {I_{rms}}^2 R$$

$$I_{rms} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

$$\Delta V_{rms} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707 \Delta V_{\max}$$

Inductors in AC Circuits



$$\Delta v - \Delta v_L = 0$$

$$\Delta V_{\max} \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{\Delta V_{\max}}{L} \sin \omega t dt$$

$$i_L = \int di = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

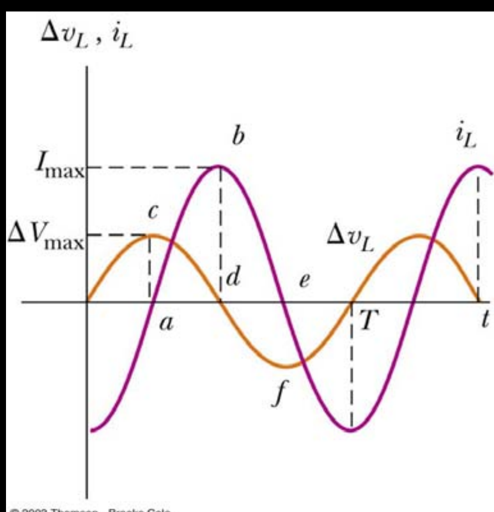
$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L} = \frac{\Delta V_{\max}}{X_L}$$

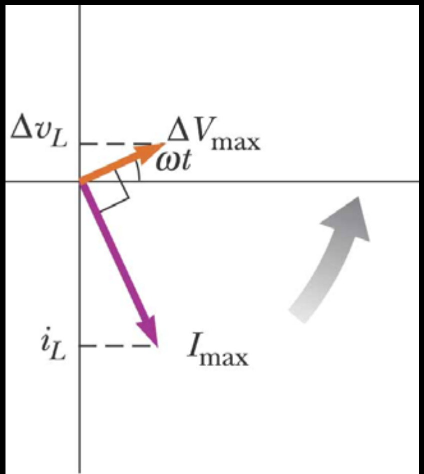
$$X_L = \omega L$$

Inductive Reactance

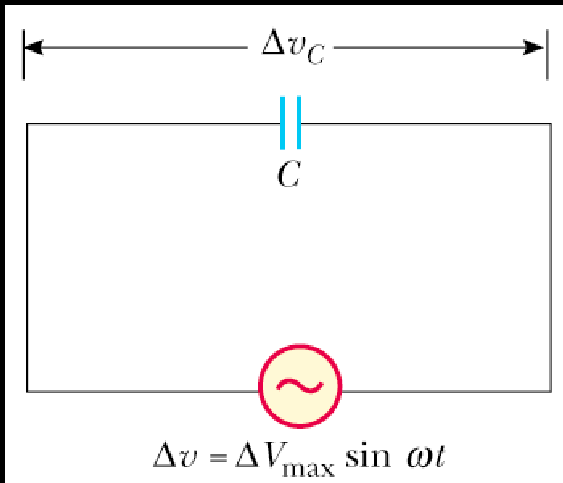
$$\Delta v_L = -I_{\max} X_L \sin \omega t$$



For a sinusoidal voltage, the current in the inductor always **lags** the voltage across it by 90°.



Capacitors in AC Circuits



$$\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$$

$$q = C\Delta V_{max} \sin \omega t$$

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

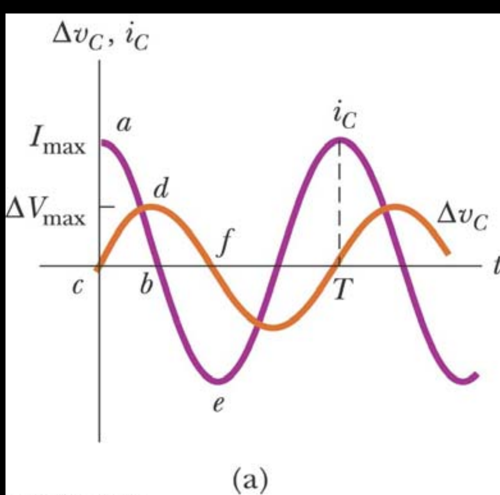
$$i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{X_C}$$

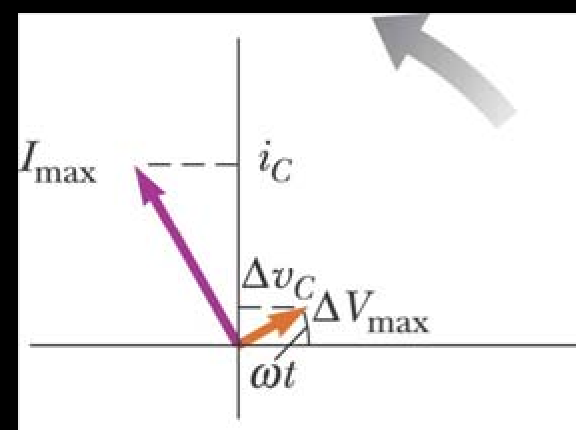
$$X_C = \frac{1}{\omega C}$$

Capacitive Reactance

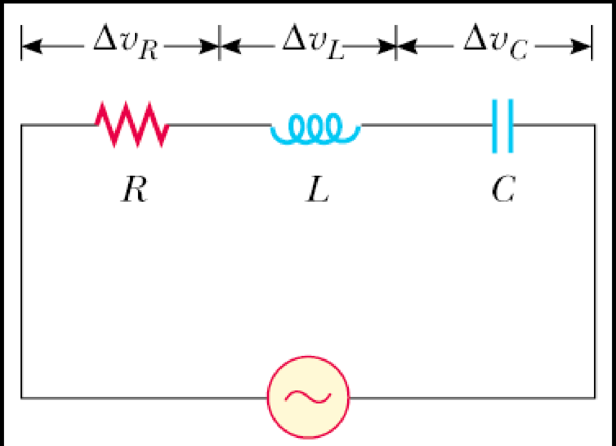
$$\Delta v_C = I_{max} X_C \sin \omega t$$

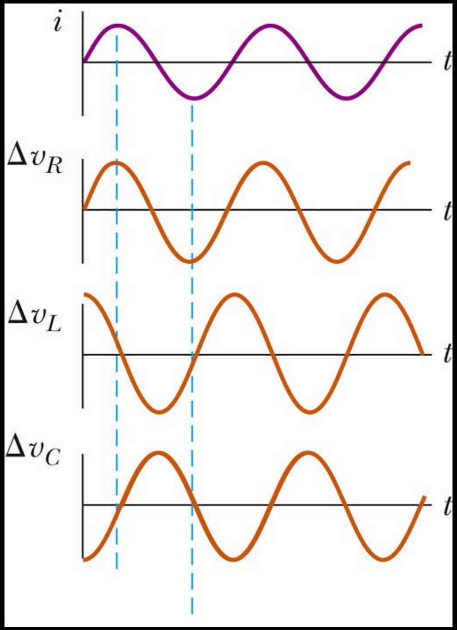


For a sinusoidal voltage, the current in the capacitor always **leads** the voltage across it by 90°.



RLC Series Circuit





$$\Delta v = \Delta V_{\max} \sin \omega t$$

$$i = I_{\max} \sin(\omega t - \phi)$$

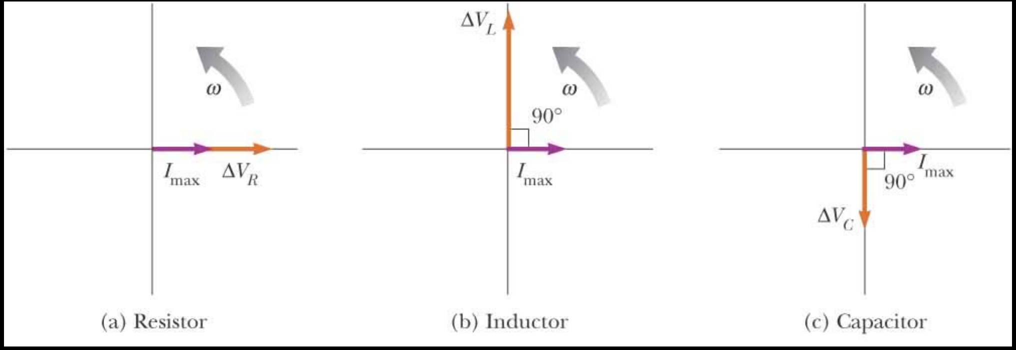
$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

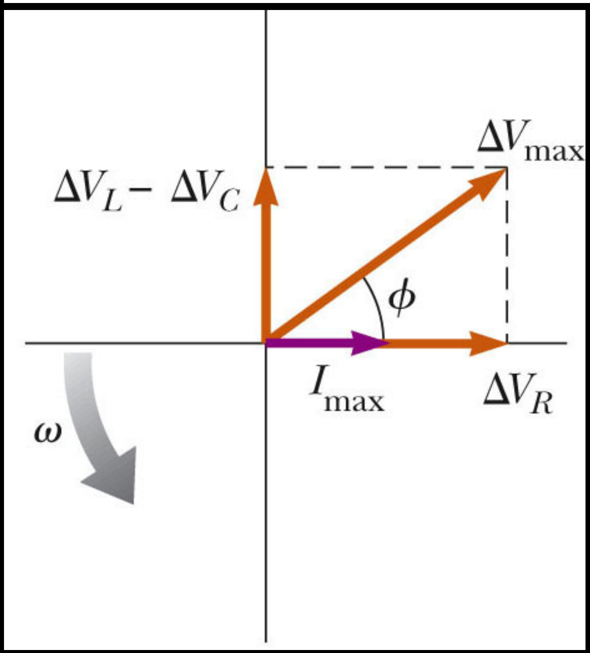
$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

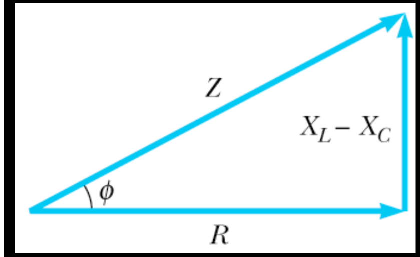
$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

Using Phasors in a RLC Circuit



$$\Delta V_{max} = \sqrt{\Delta V_R{}^2 + (\Delta V_L - \Delta V_C)^2}$$
$$\Delta V_{max} = \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$
$$\Delta V_{max} = I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$





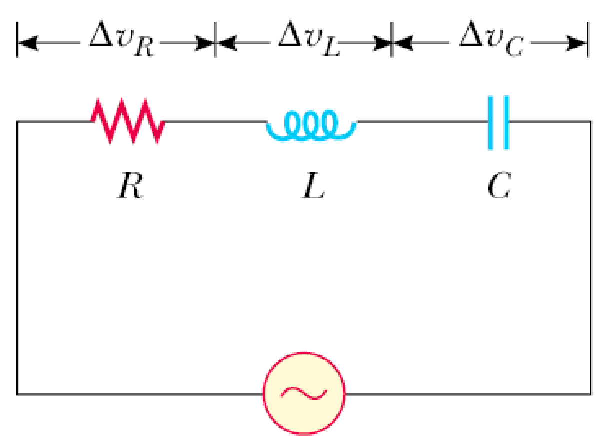
$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$ Impedance

$$\Delta V_{max} = I_{max} Z$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Example 33.4



R = 425 Ω
L = 1.25 H
C = 3.5 μF
ω = 377 s^{-1}
ΔV_{max} = 150 V

$$X_L = \omega L = (377 s^{-1})(1.25H) = 471\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 s^{-1})(3.5\mu F)} = 758\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(425\Omega)^2 + (471\Omega - 758\Omega)^2} = 513\Omega$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150V}{513\Omega} = 0.292A$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\Omega - 758\Omega}{425\Omega}\right) = -34°$$

$$\Delta V_R = I_{\max} R = (0.292A)(425\Omega) = 124V$$

$$\Delta V_L = I_{\max} X_L = (0.292A)(471\Omega) = 138V$$

$$\Delta V_C = I_{\max} X_C = (0.292A)(758\Omega) = 221V$$

$$\Delta v_R = (124V)\sin 377t$$

$$\Delta v_L = (138V)\cos 377t$$

$$\Delta v_C = -(221V)\cos 377t$$

Power in an AC Circuit

$$\mathcal{P} = i\Delta v = I_{\max}\sin(\omega t - \phi)\Delta V_{\max}\sin\omega t$$

$$\mathcal{P} = I_{\max}\Delta V_{\max}(\sin\omega t\cos\phi - \cos\omega t\sin\phi)\sin\omega t$$

$$\mathcal{P} = I_{\max}\Delta V_{\max}\sin^2\omega t\cos\phi - I_{\max}\Delta V_{\max}\sin\omega t\cos\omega t\sin\phi$$

$$\mathcal{P}_{av} = \frac{1}{2}I_{\max}\Delta V_{\max}\cos\phi$$

$$\mathcal{P}_{av} = I_{rms}\Delta V_{rms}\cos\phi$$

$$\Delta v_R = \Delta V_{\max}\cos\phi = I_{\max}R$$

$$\mathcal{P}_{av} = I_{rms}\left(\frac{\Delta V_{\max}}{\sqrt{2}}\right)\frac{I_{\max}R}{\Delta V_{\max}} = I_{rms}\frac{I_{\max}R}{\sqrt{2}}$$

$$\mathcal{P}_{av} = I_{rms}{}^2 R$$

No power loss occurs in an ideal capacitor or inductor.

For a purely resistive load, $\phi=0$

$$\mathcal{P}_{av} = I_{rms}\Delta V_{rms}$$

Resonance in Series RLC Circuits

A circuit is in resonance when its current is maximum.

$$I_{rms} = \frac{\Delta V_{rms}}{Z}$$

$$I_{rms} = \frac{\Delta V_{rms}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

I_{rms}=*max* when X_L-X_C=0

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance Frequency

Power in Series RLC Circuits

$$\mathcal{P}_{av} = I_{rms}^{\ 2} R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2}$$

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}\left(\omega^2 - \omega_0^{\ 2}\right)^2$$

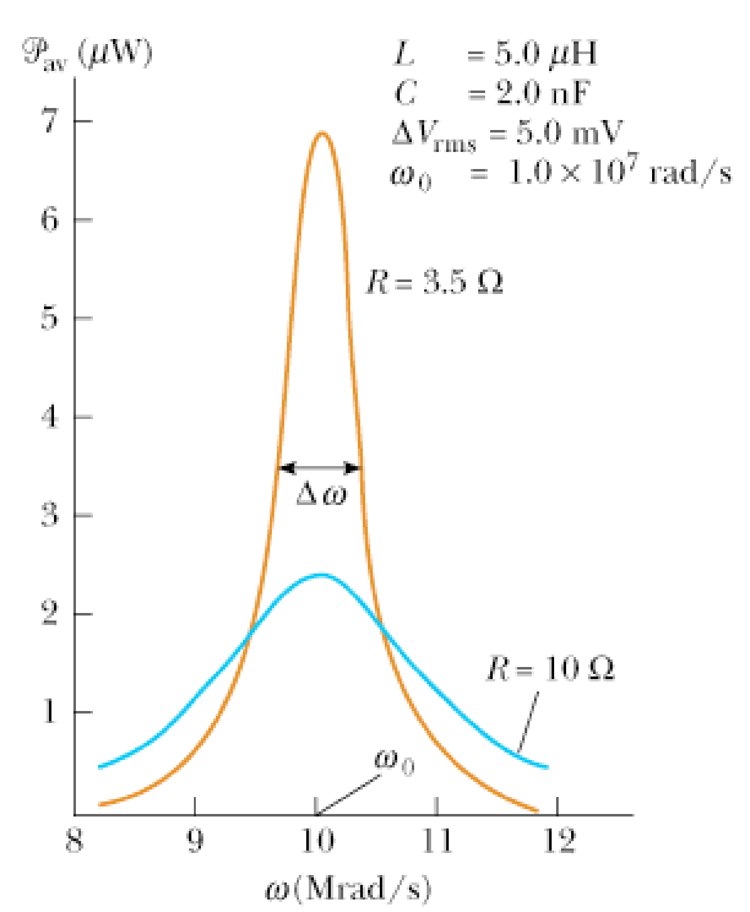
$$\mathcal{P}_{av} = \frac{(\Delta V_{rms})^2 R \omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_0^{\ 2}\right)^2}$$

$$\mathcal{P}_{av,\max} = \frac{(\Delta V_{rms})^2}{R}$$

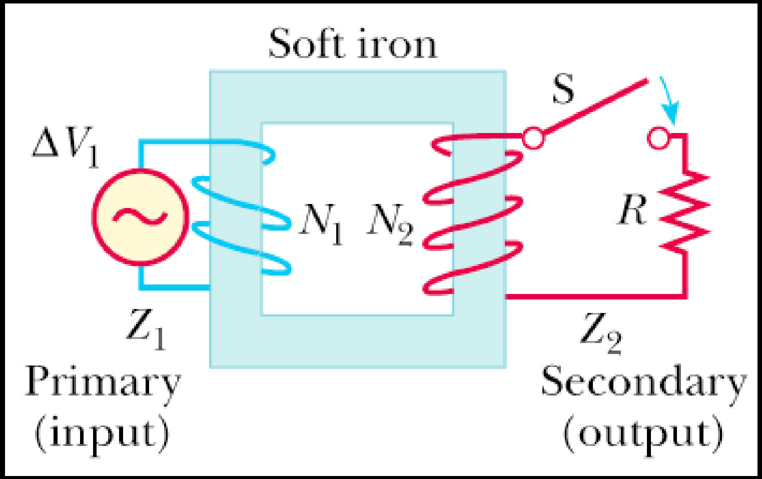
$\omega = \omega_0$

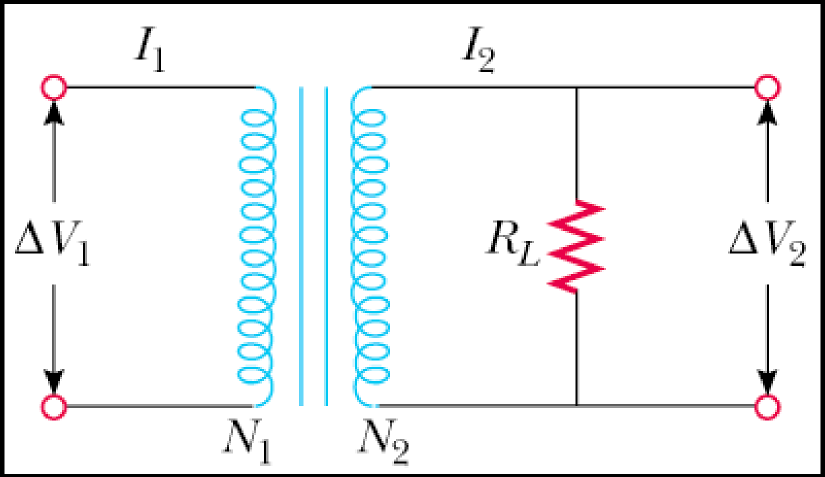
$$\mathcal{Q} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}$$

Quality Factor



Transformers





$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt}$$

$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$

$$I_1 \Delta V_1 = I_2 \Delta V_2$$

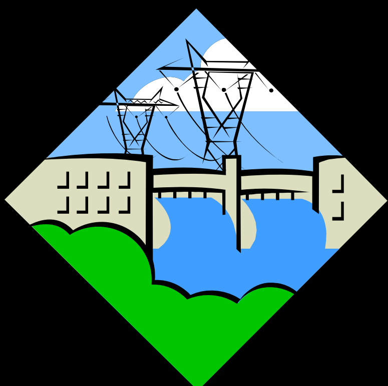
$$I_2 = \frac{\Delta V_2}{R_L}$$

$$I_1 = \frac{\Delta V_1}{R_{eq}}$$

$$R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

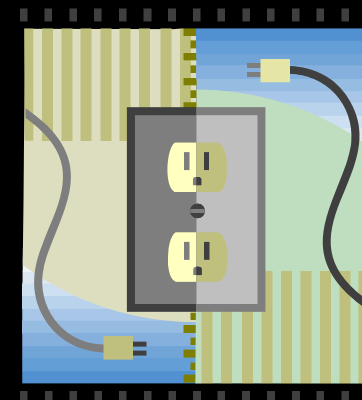
Power Transmission

Power is transmitted in high voltage over long distances and then gradually stepped down to 240V by the time it gets to a house.



22 kV

230 kV



240 V

$\mathcal{P}_g$ = 20 MW
Cost = 10¢/kWh

$$I = \frac{\mathcal{P}_g}{\Delta V} = \frac{20 \times 10^6 W}{230 \times 10^3 V} = 87A$$

$$\mathcal{P} = I^2 R = (87A)^2 (2\Omega) = 15kW$$

$$TotalCost = (15kW)(24h)(\$0.1) = \$36$$

For 22kV transmission:

Total Cost = $4100

For Next Class

- Midterm 2 Review on Monday
- Midterm 2 on Tuesday
- Reading Assignment for Wednesday
  - Chapter 34 – Electromagnetic Waves
- WebAssign: Assignment 11 (due Monday, 5 PM)