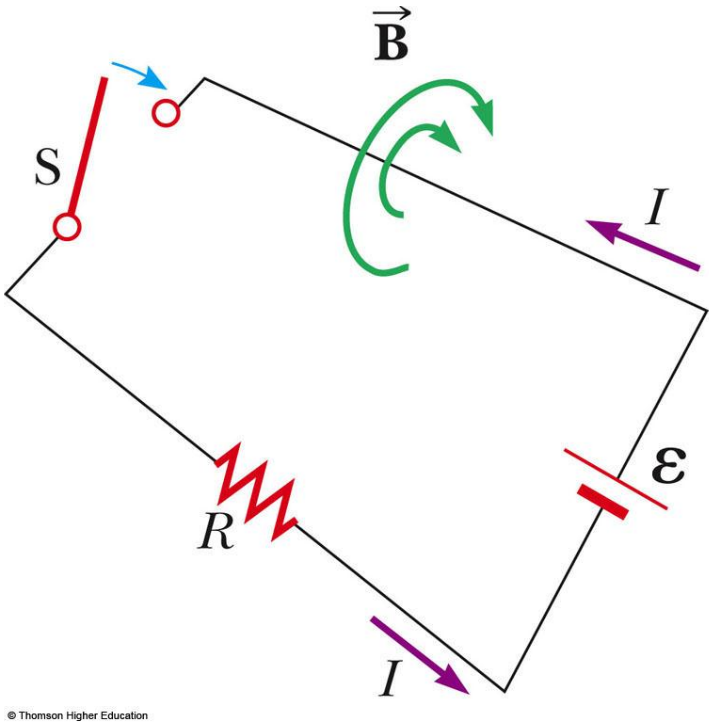
Inductance

- Self-Inductance
- RL Circuits
- Energy in a Magnetic Field
- Mutual Inductance
- Oscillations in an LC Circuit

Self-Inductance



When the switch is closed, the battery (source emf) starts pushing electrons around the circuit.

The current starts to rise, creating an increasing magnetic flux through the circuit.

This increasing flux creates an induced emf in the circuit.

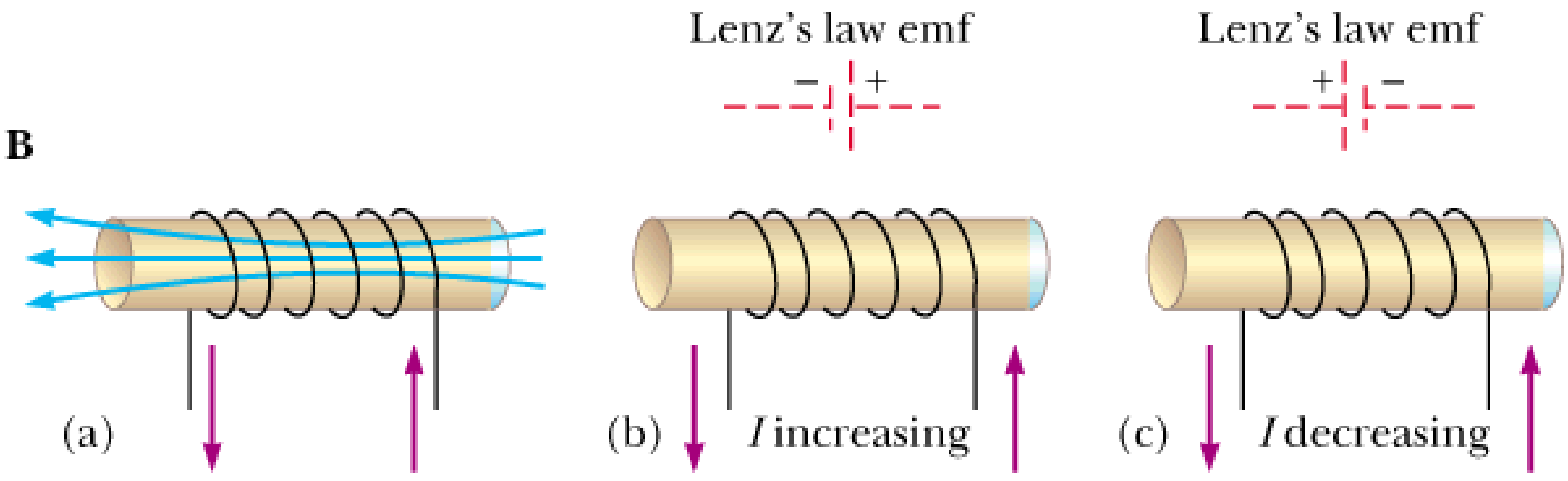
The induced emf will create a flux to oppose the increasing flux.

The direction of this induced emf will be opposite the source emf.

This results in a gradual increase in the current rather than an instantaneous one.

The induced emf is called the **self-induced emf** or **back emf**.

Current in a Coil



When *I* changes, an emf is induced in the coil.

If *I* is increasing (and therefore increasing the flux through the coil), then the induced emf will set up a magnetic field to oppose the increase in the magnetic flux in the direction shown.

If *I* is decreasing, then the induced emf will set up a magnetic field to oppose the decrease in the magnetic flux.

Self-Inductance

$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt}$$

$$\Phi_B \propto B$$

$$B \propto I$$

$$\mathcal{E}_L \propto \frac{dI}{dt}$$

$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

$$L = \frac{N\Phi_B}{I}$$

(Henry=H=V.s/A)

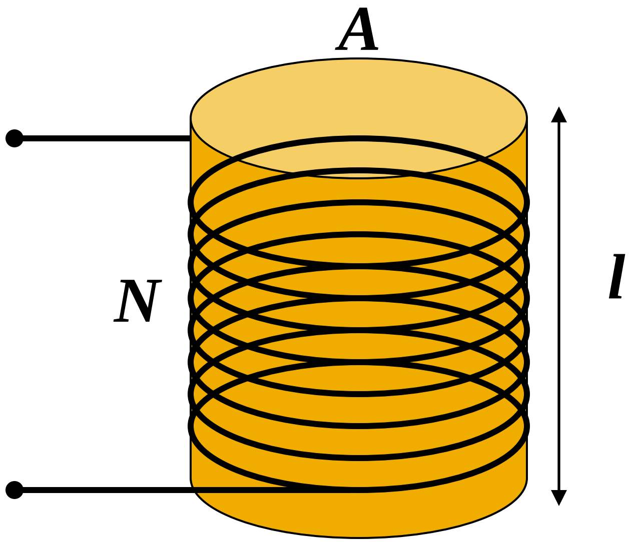
$$L = -\frac{\mathcal{E}_L}{dI/dt}$$

Inductance of a Solenoid

$$B = \mu_0 n I = \mu_0 \frac{N}{l} I$$

$$\Phi_B = BA = \mu_0 \frac{NA}{l} I$$

$$L = \frac{N\Phi_B}{I} = \mu_0 \frac{N^2 A}{l}$$



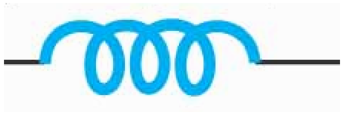
$$L = \mu_0 n^2 V$$

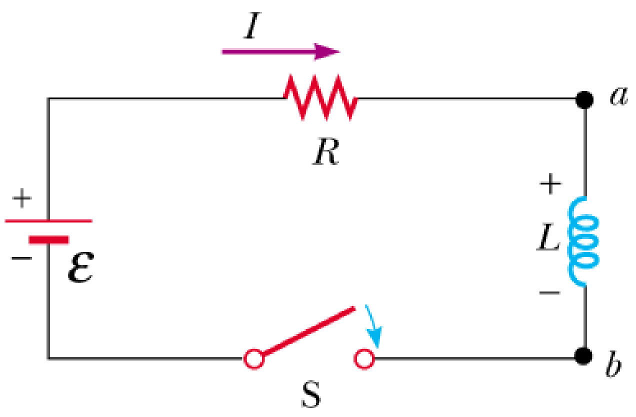
$$N = nl$$

$$V = Al$$

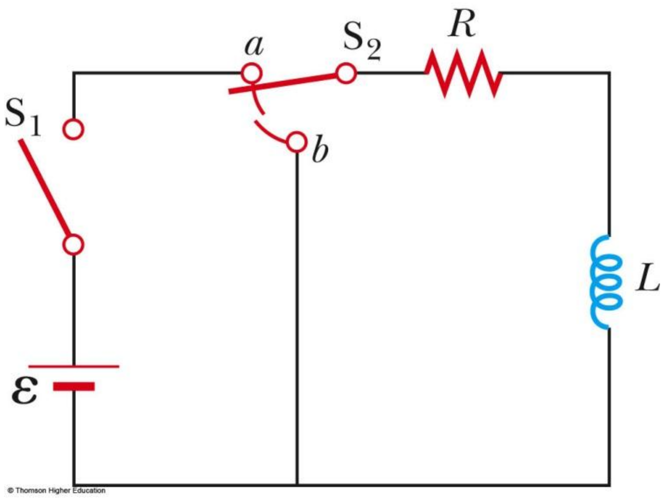
RL Circuits

- Inductors are circuit elements with large self-induction.
- A circuit with an inductor will generate some back-emf in response to a changing current.
- This back-emf will act to keep the current the way it used to be.
- Such a circuit will act "sluggish" in its response.





RL Circuit Analysis

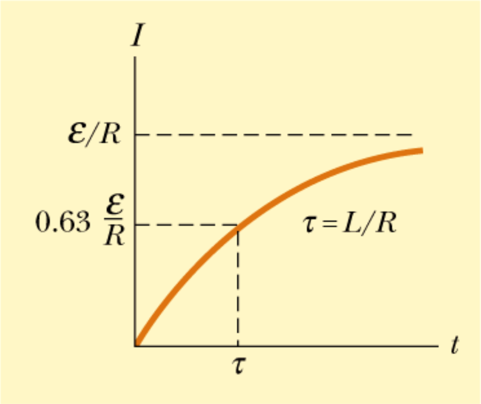


$$\mathcal{E}_L = -L\frac{dI}{dt}$$

When S_2 is at *a*

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R}$$



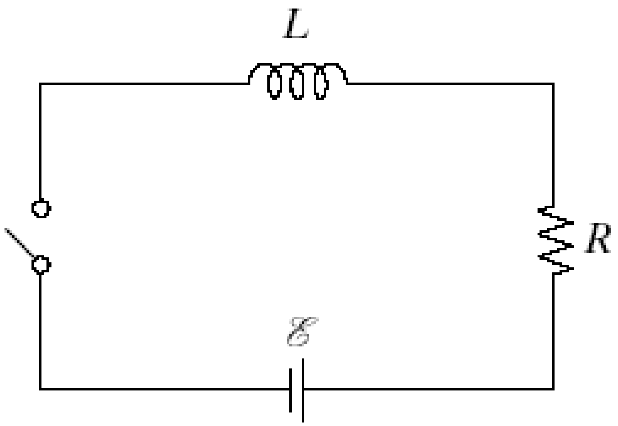
When S_2 is at *b*

$$IR + L\frac{dI}{dt} = 0$$

$$I = I_i e^{-t/\tau} \qquad \tau = \frac{L}{R}$$

Concept Question

When the switch is closed, the current through the circuit exponentially approaches a value $I = \mathcal{E} / R$. If we repeat this experiment with an inductor having twice the number of turns per unit length, the time it takes for the current to reach a value of $I / 2$



1. increases.
2. decreases.
3. is the same.

Energy in a Magnetic Field

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

Battery Power

Resistor Power

Inductor Power

The energy stored in the inductor

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$U = \int dU = \int_0^I LIdI = L\int_0^I IdI$$

$$U = \frac{1}{2}LI^2$$

For energy density, consider a solenoid

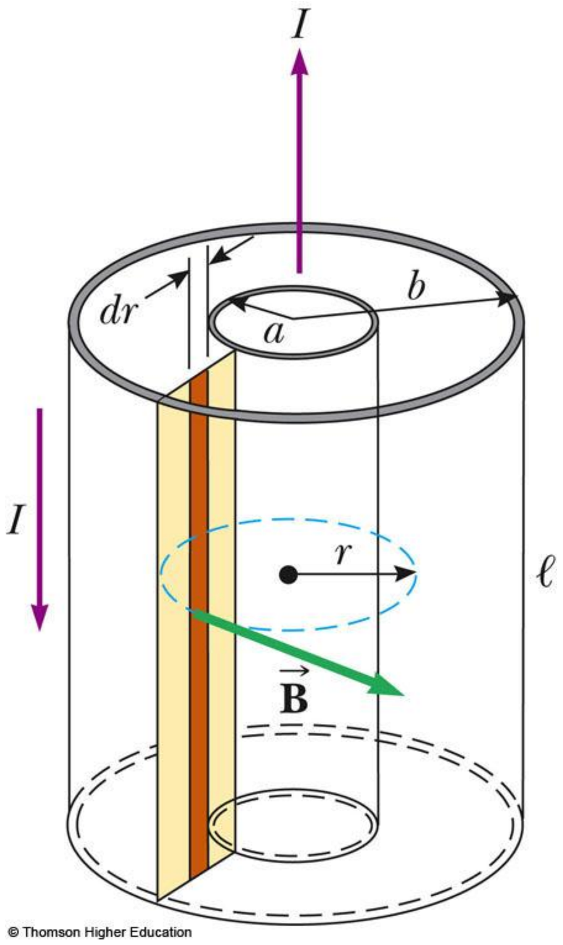
$$L = \mu_0 n^2 Al$$

$$B = \mu_0 nI$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 Al\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}Al$$

$$u_B = \frac{U}{Al} = \frac{B^2}{2\mu_0}$$

Inductance of a Coaxial Cable



$$\Phi_B = \int B dA$$

$$dA = l dr$$

$$\Phi_B = \int B dA = \int_a^b \frac{\mu_0 I}{2\pi r} l dr = \frac{\mu_0 I l}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

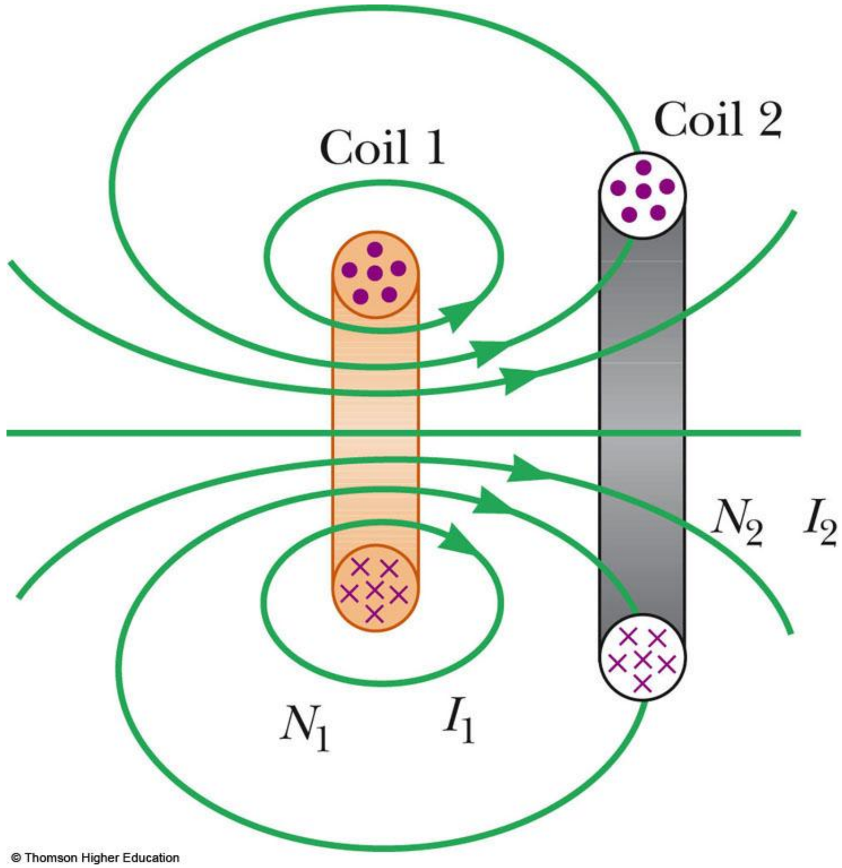
$$L = \frac{\Phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$U = \frac{1}{2} L I^2$$

$$U = \frac{\mu_0 l I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

Mutual Inductance

A change in the current of one circuit can induce an emf in a nearby circuit



$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

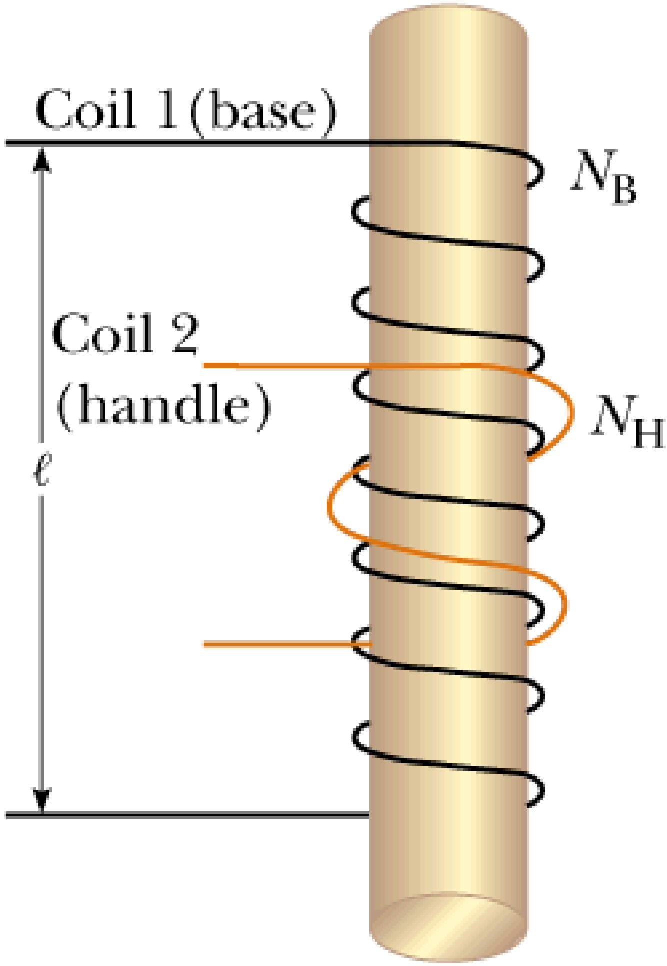
$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12} \frac{dI_1}{dt}$$

$$\mathcal{E}_1 = -M_{21} \frac{dI_2}{dt}$$

It can be shown that: $M_{12} = M_{21} = M$

$$\mathcal{E}_1 = -M \frac{dI_2}{dt} \qquad \mathcal{E}_2 = -M \frac{dI_1}{dt}$$

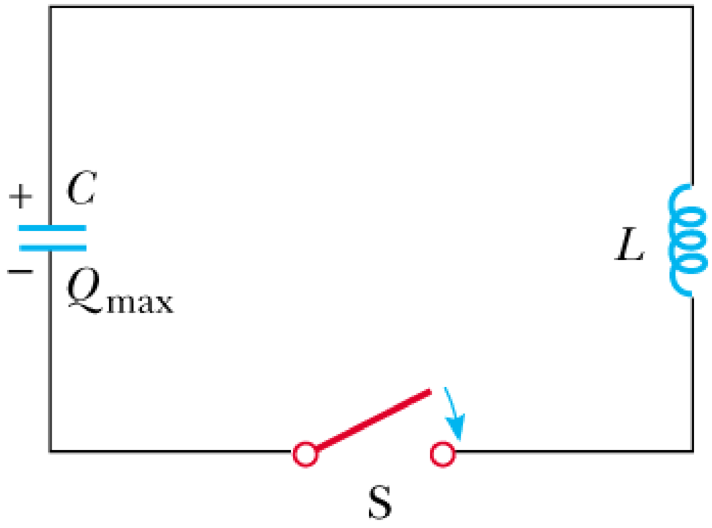
Wireless Battery Charger



$$B = \mu_0 \frac{N_B}{l} I$$

$$M = \frac{N_H \Phi_{BH}}{I} = \frac{N_H BA}{I} = \mu_0 \frac{N_H N_B A}{l}$$

LC Oscillators



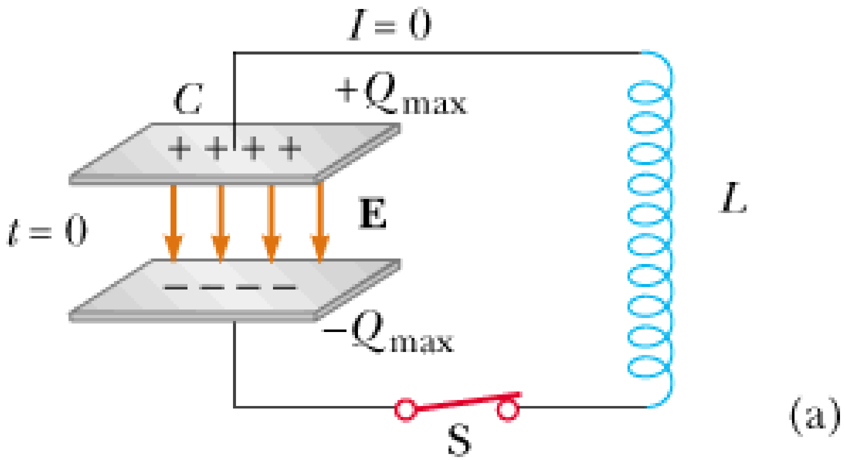
- Provide a transfer of energy between the capacitor and the inductor.
- Analogous to a spring-block system

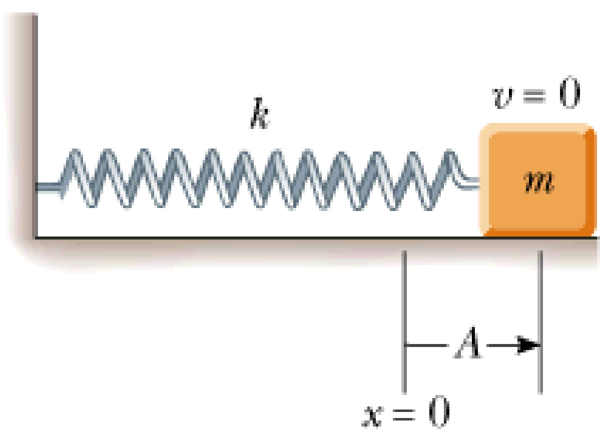
The Oscillation Cycle

Assume the capacitor is fully charged and thus has some stored energy.

When the switch is closed, the charges on the capacitor leave the plates and move in the circuit, setting up a current.

This current starts to discharge the capacitor and reduce its stored energy.

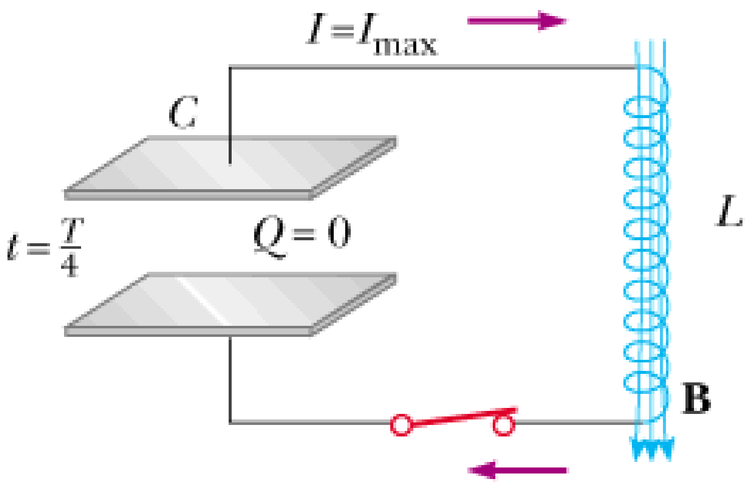




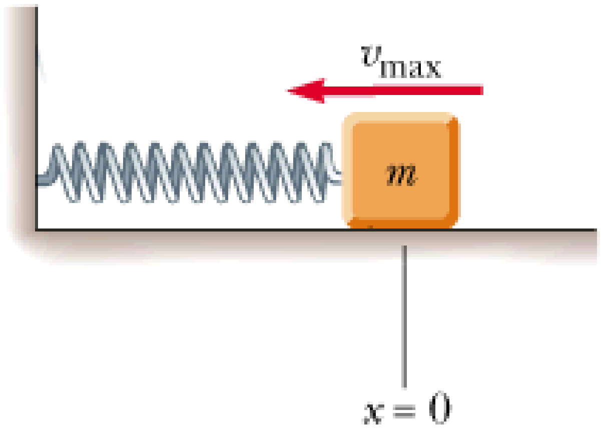
The Oscillation Cycle

At the same time, the current increases the stored energy in the magnetic field of the inductor.

When the capacitor is fully discharged, it stores no energy, but the current reaches a maximum and all the energy is stored in the inductor.



(b)

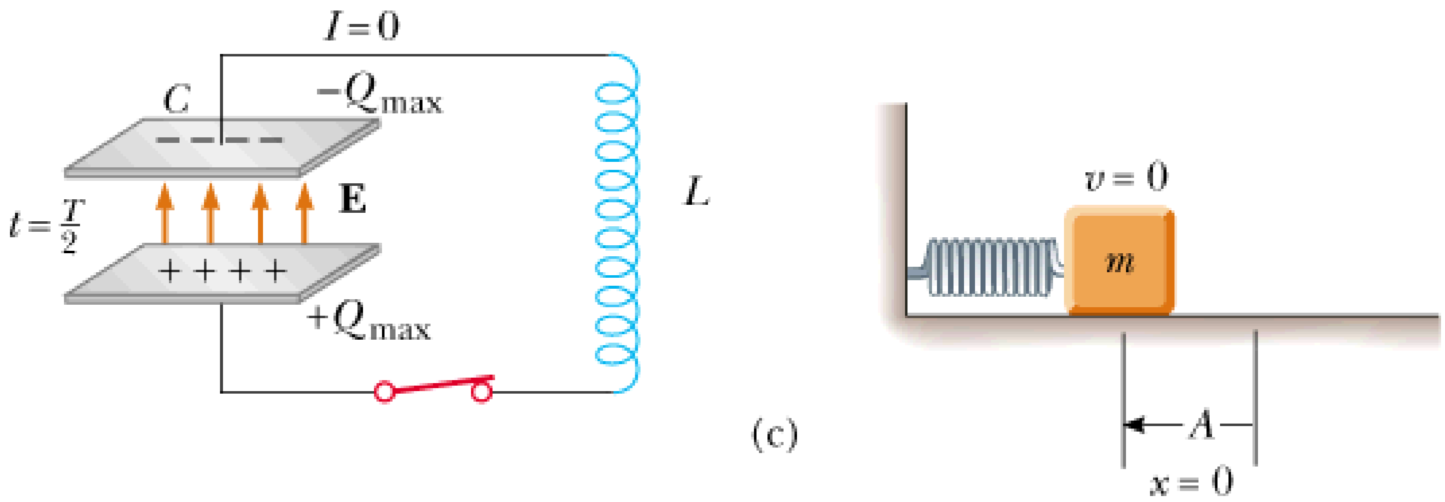


The Oscillation Cycle

The current continues to flow and now starts to charge the capacitor again, this time with the opposite polarity.

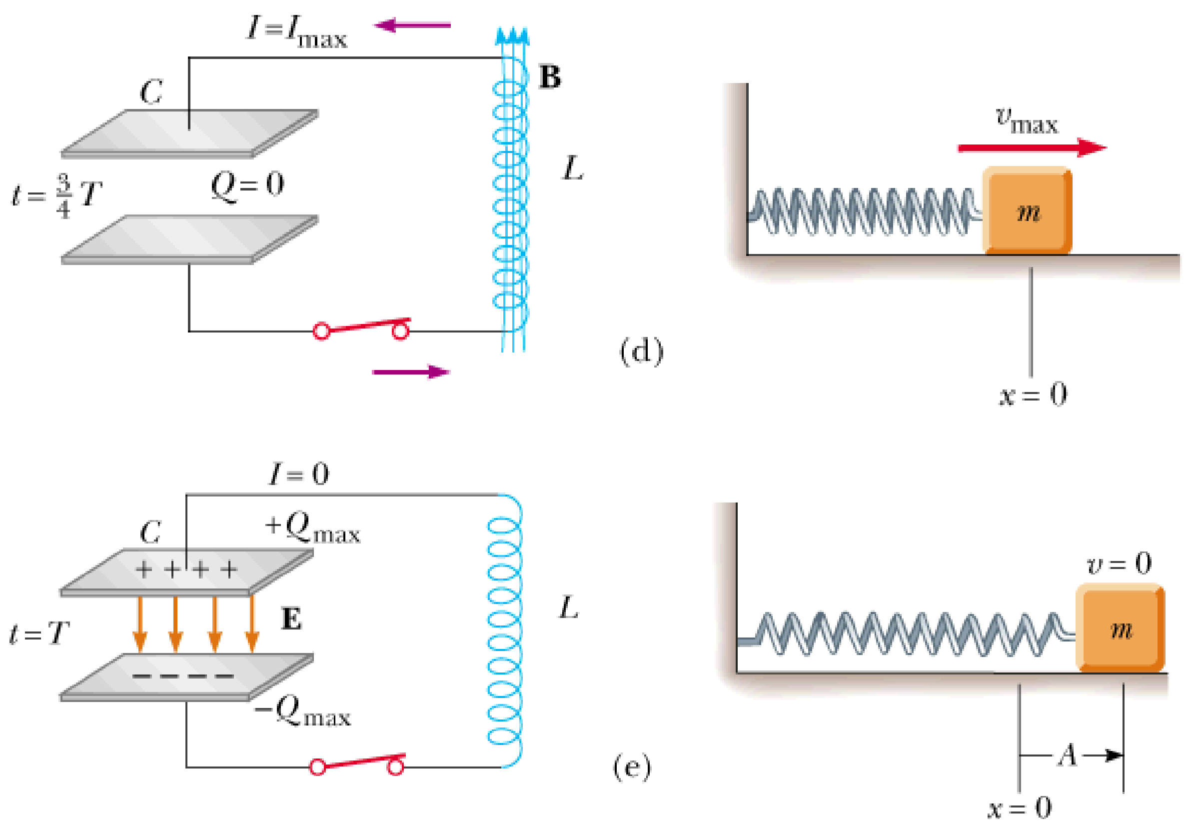
When the capacitor becomes fully charged (with the opposite polarity), the energy is completely stored in the capacitor again.

The e-field sets up the current flow in the opposite direction.



The Oscillation Cycle

Then the cycle completes itself in reverse.



Charge and Current in LC Circuits

At some arbitrary time, t:

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

$$\frac{dU}{dt} = 0$$

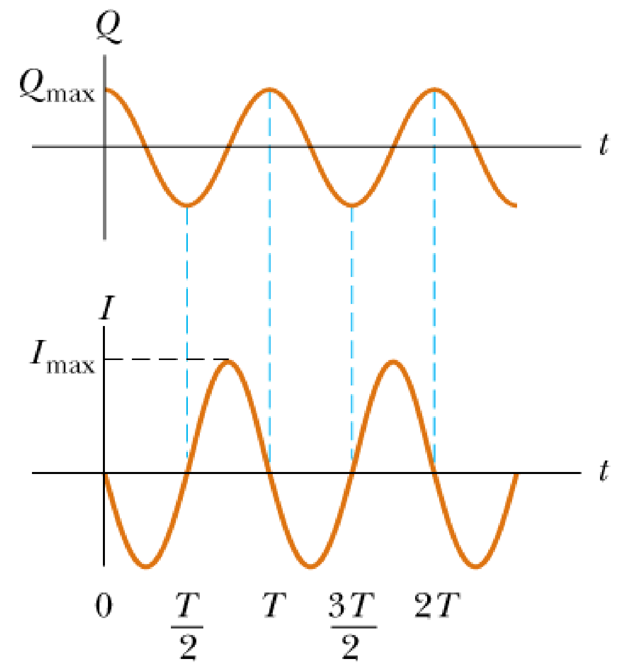
$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$Q = Q_{max}\cos\omega t$$

$$I = -\omega Q_{max}\sin\omega t = -I_{max}\sin\omega t$$

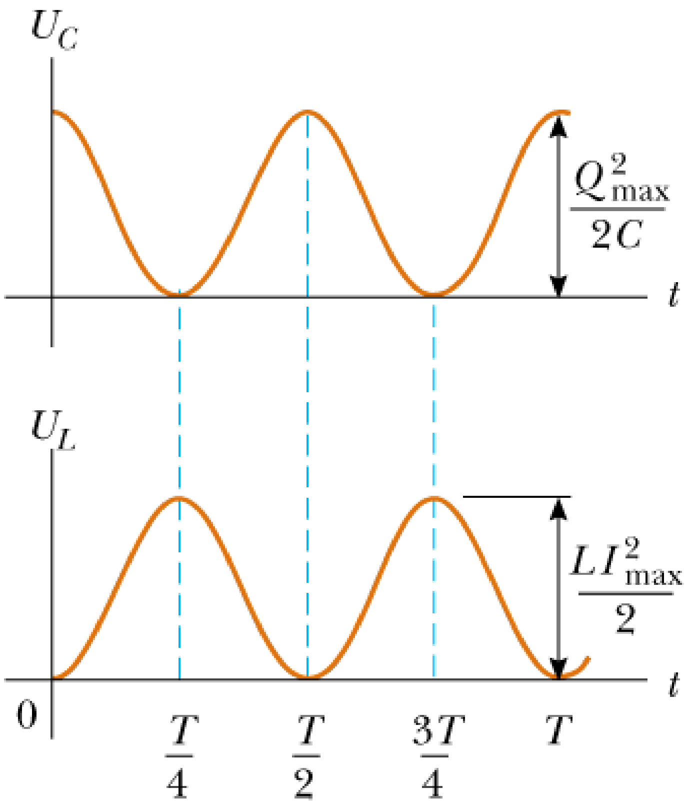
$$\omega = \frac{1}{\sqrt{LC}}$$

Natural frequency of oscillation



Energy in LC Circuits

$$U = U_C + U_L = \frac{Q_{\max}^2}{2C}\cos^2 \omega t + \frac{LI^2_{\max}}{2}\sin^2 \omega t$$

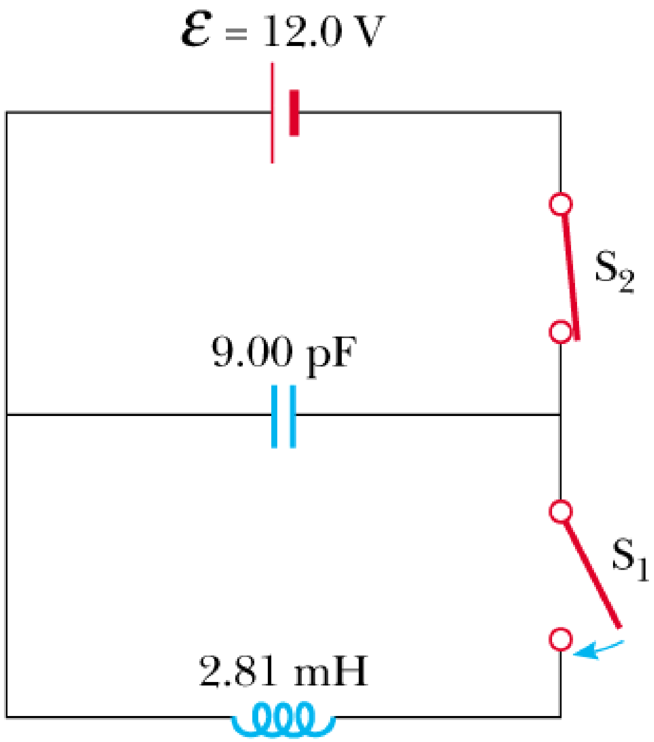


$$\frac{Q_{\max}^2}{2C} = \frac{LI^2_{\max}}{2}$$

$$U = \frac{Q_{\max}^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right)$$

$$U = \frac{Q_{\max}^2}{2C}$$

Oscillations in an LC Circuit



a) $f = ?$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \frac{1}{2\pi\sqrt{(2.81\times10^{-3})(9\times10^{-12})}} = 10^{6}\,Hz$$

b) $Q_{max} = ?$ and $I_{max} = ?$

$$Q_{\max} = C\mathcal{E} = (9\times10^{-12})(12) = 1.08\times10^{-10}C$$

$$I_{\max} = \omega Q_{\max} = 2\pi f Q_{\max} = 6.79\times10^{-4}A$$

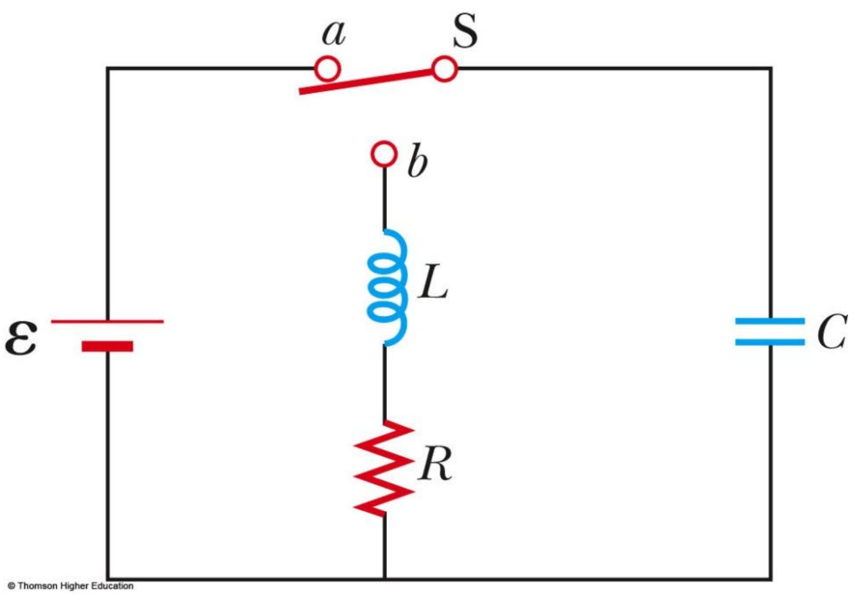
c) $Q(t) = ?$ and $I(t) = ?$

$$Q = Q_{\max}\cos\omega t = (1.08\times10^{-10}C)\cos(2\pi\times10^{6}t)$$

$$I = -I_{\max}\sin\omega t = -(6.79\times10^{-4}A)\sin(2\pi\times10^{6}t)$$

d) $U = ?$ $$U = \frac{Q_{\max}{}^{2}}{2C} = 6.48\times10^{-10}J$$

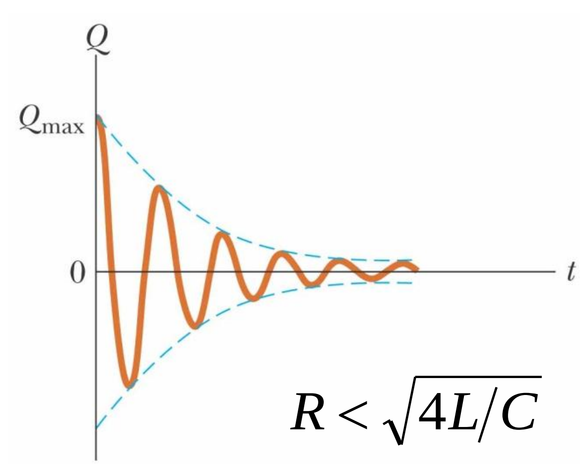
RLC Circuits – Damped Oscillations



$$Q = Q_{\max} e^{-Rt/2L} \cos \omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L} \right)^2 \right]^{1/2}$$

- RLC Circuit - A more realistic circuit
- The resistor represents the losses in the system.
- Can be oscillatory, but the amplitude decreases.



For Next Class

- Reading Assignment
  - Chapter 33 - Alternating Current Circuits
- WebAssign: Assignment 10