Electromagnetic Waves

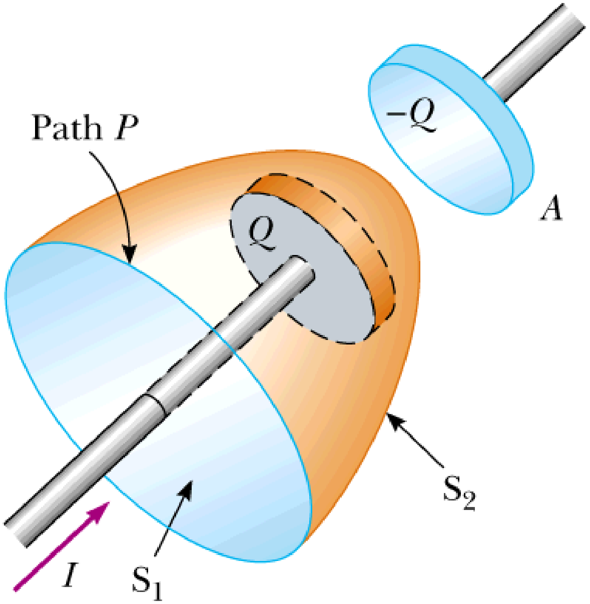
- Ampere's Law Updated
- Hertz's Discoveries
- Plane Electromagnetic Waves
- Energy in an EM Wave
- Momentum and Radiation Pressure
- The Electromagnetic Spectrum

Problem with Ampère's Law

Ampère's Law $$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I$$

But there is a problem if *I* varies with time!

Consider a parallel plate capacitor



For S_1, $$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I$$

For S_2, $$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I = 0$$

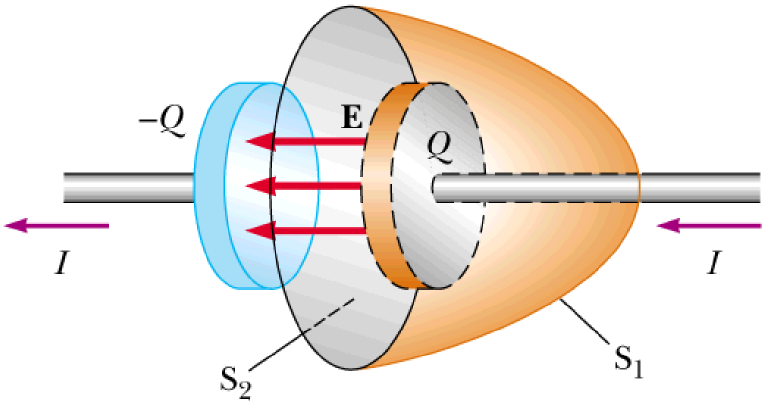
(No current in the gap)

Solution - Displacement Current

Call *I* as the **conduction** current.

Specify a new current that exists when a change in the electric field occurs. Call it the **displacement** current.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$



Electrical flux through S_2:

$$\Phi_E = EA = \left(\frac{Q}{\varepsilon_0 A}\right)A = \frac{Q}{\varepsilon_0}$$

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I$$

Ampère-Maxwell Law

Now the generalized form of Ampère's Law, or Ampère-Maxwell Law becomes:

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 \left(I + I_d \right) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Magnetic fields are produced by both conduction currents and by time-varying electrical fields.

Maxwell's Equations

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$$

Gauss' Law

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{A}} = 0$$

Gauss' Law for Magnetism – no magnetic monopoles

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

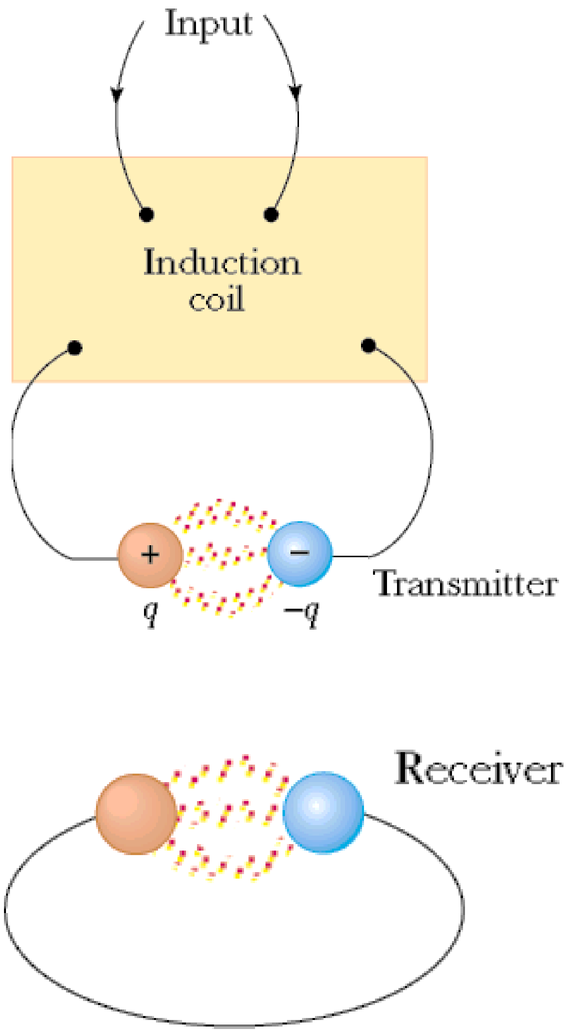
$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampère-Maxwell Law

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Lorentz Force Law

Hertz's Radio Waves



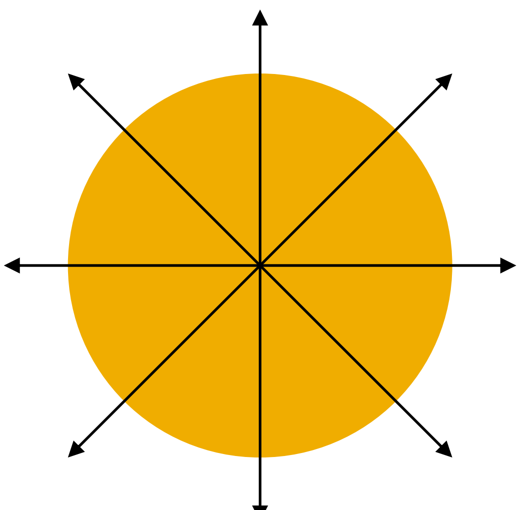
By supplying short voltage bursts from the coil to the transmitter electrode we can ionize the air between the electrodes.

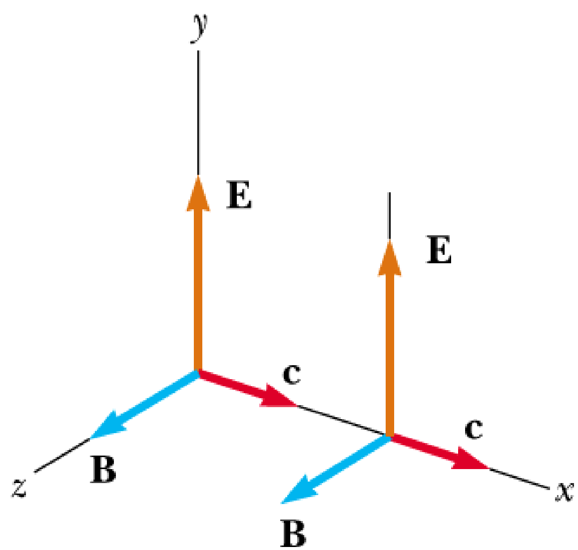
In effect, the circuit can be modeled as a LC circuit, with the coil as the inductor and the electrodes as the capacitor.

A receiver loop placed nearby is able to receive these oscillations and creates sparks as well.

Waves

Circular (or Spherical in 3D) Waves





Plane Waves

Electric and Magnetic Fields in Free Space

$$\oint \vec{\mathbf{E}}.d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Free Space: $I = 0$, $Q = 0$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

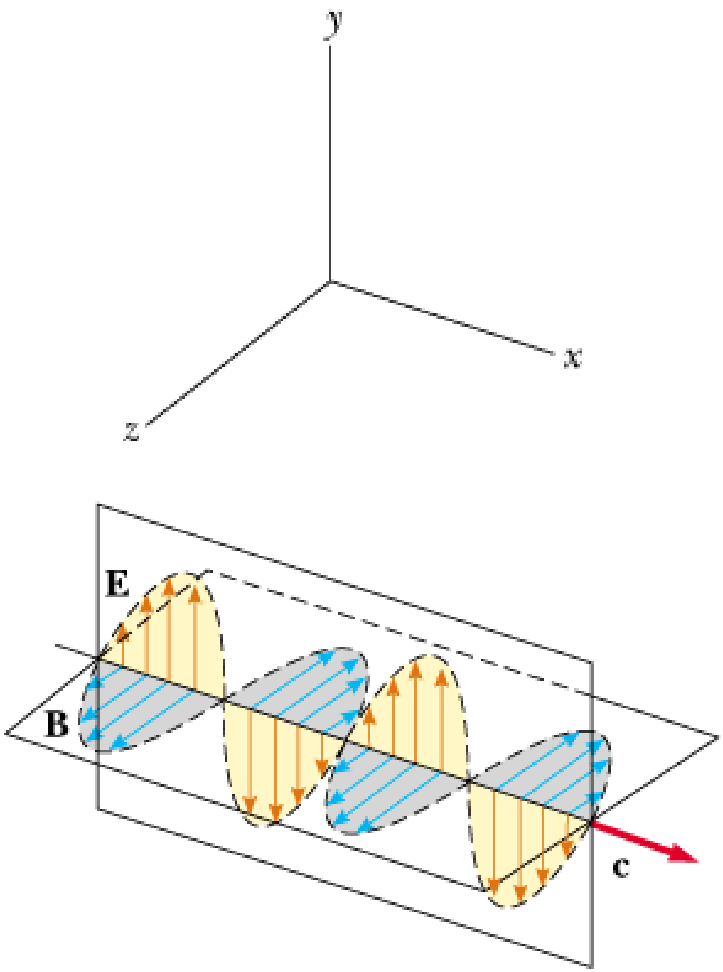
$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

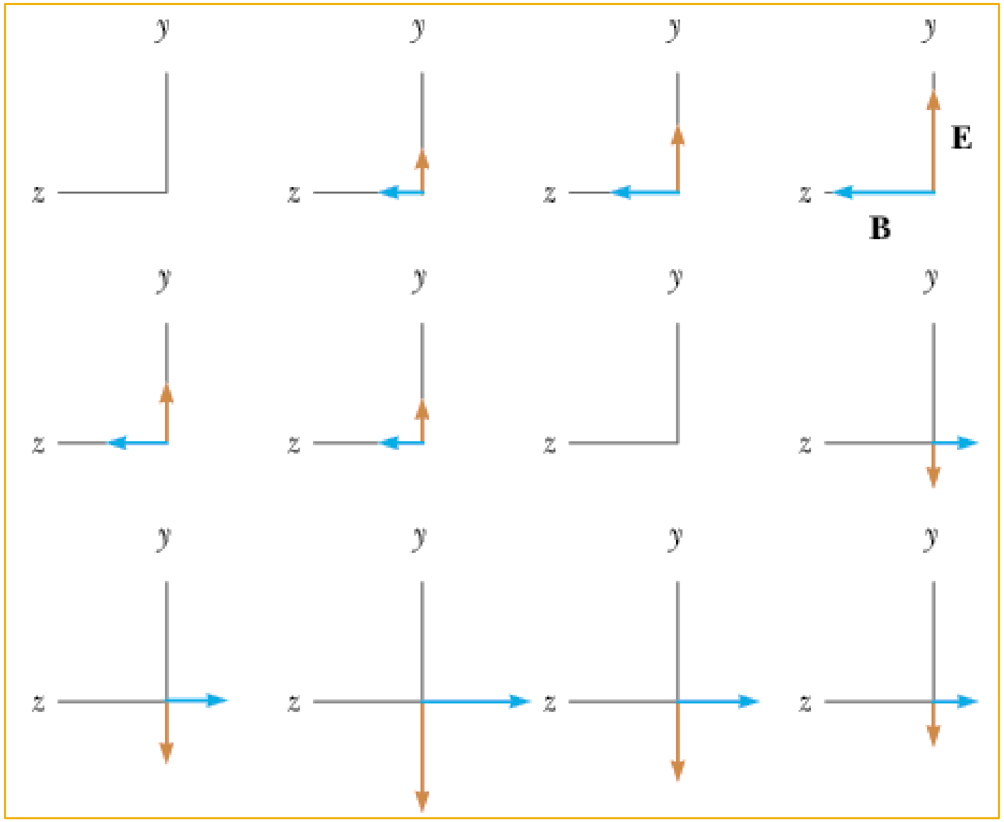
$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$

EM Wave Oscillation





Some Important Quantities

$$E = E_{max} \cos(kx - \omega t)$$
$$B = B_{max} \cos(kx - \omega t)$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ Speed of Light

$$\omega = 2\pi f$$ Angular Frequency

$$\lambda = \frac{c}{f}$$ Wavelength

$$k = \frac{2\pi}{\lambda}$$ Wavenumber

$$\frac{\omega}{k} = c$$

Properties of EM Waves

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$E = E_{\max} \cos(kx - \omega t)$$

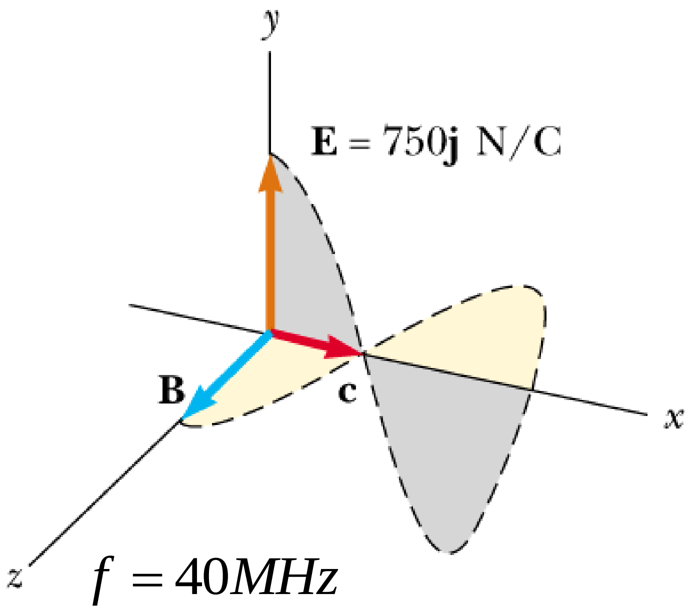
$$B = B_{\max} \cos(kx - \omega t)$$

$$kE_{\max} = \omega B_{\max}$$

$$\frac{E}{B} = \frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = c$$

- The solutions to Maxwell's equations in free space are wavelike
- Electromagnetic waves travel through free space at the speed of light.
- The electric and magnetic fields of a plane wave are perpendicular to each other and the direction of propagation (they are transverse).
- The ratio of the magnitudes of the electric and magnetic fields is c.
- EM waves obey the superposition principle.

An EM Wave



$$f = 40MHz$$

a) *λ=?, T=?*

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{40\times10^6} = 7.5m$$

$$T = \frac{1}{f} = \frac{1}{40\times10^6} = 2.5\times10^{-8}s$$

b) If *E_{max}=750N/C,* then *B_{max}=?*

$$B_{\max} = \frac{E_{\max}}{c} = \frac{750}{3\times10^8} = 2.5\times10^{-6}T$$

Directed towards the z-direction

c) *E(t)=?* and *B(t)=?*

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = 0.838 rad/m$$

$$\omega = 2\pi f = 2\pi 40\times10^6 = 2.51\times10^8 rad/s$$

$$E = E_{\max}\cos(kx-\omega t) = (750N/C)\cos(0.838x - 2.51\times10^8 t)$$

$$B = B_{\max}\cos(kx-\omega t) = (2.5\times10^{-6}T)\cos(0.838x - 2.51\times10^8 t)$$

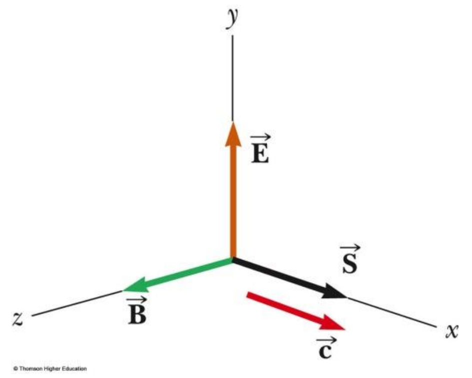
Energy Carried by EM Waves

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

Poynting Vector (W/m^2)

S is equal to the rate of EM energy flow per unit area (power per unit area)

For a plane EM Wave: $$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{c}{\mu_0}B^2$$



The average value of S is called the **intensity**:

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^{\;2}}{2\mu_0 c} = \frac{c}{2\mu_0}B_{max}^{\;2}$$

The Energy Density

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}$$

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0 \varepsilon_0}{2\mu_0} E^2 = \frac{1}{2}\varepsilon_0 E^2$$

For an EM wave, the instantaneous electric and magnetic energies are equal.

$$u_E = u_B = \frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Total Energy Density of an EM Wave

$$u_{av} = \varepsilon_0 \left(E^2\right)_{av} = \frac{1}{2}\varepsilon_0 E_{\max}{}^2 = \frac{B_{\max}{}^2}{2\mu_0}$$

$$I = S_{av} = c u_{av}$$

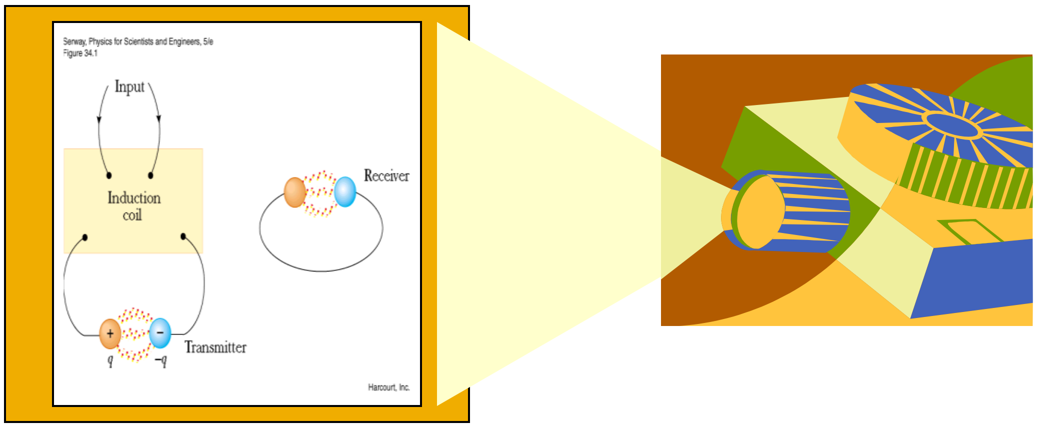
The intensity is *c* times the total average energy density

Concept Question

Which gives the largest average energy density at the distance specified and thus, at least qualitatively, the best illumination

1. a 50-W source at a distance *R.*
2. a 100-W source at a distance 2*R.*
3. a 200-W source at a distance 4*R.*

Fields on the Screen



$\mathcal{P}_{lamp}$ **= 150 W, 3% efficiency**

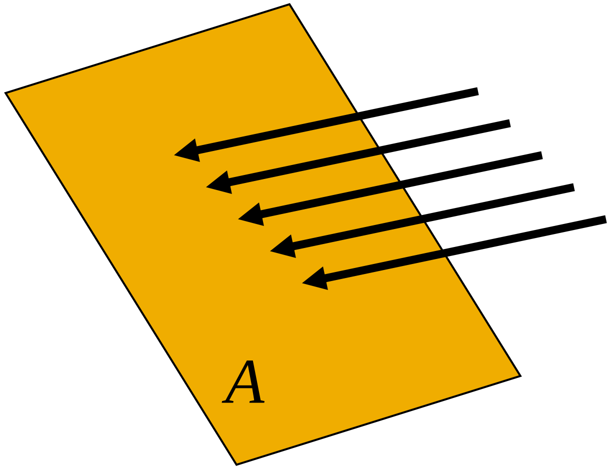
$A = 15\ m^2$

$$\mathcal{P}_{av} = \mathcal{P}_{lamp} 0.03 = 4.5W$$

$$I = \frac{\mathcal{P}_{av}}{A} = \frac{E_{\max}{}^2}{2\mu_0 c} \longrightarrow E_{\max} = \sqrt{\frac{2\mu_0 c \mathcal{P}_{av}}{A}} = 18.42V/m$$

$$B_{\max} = \frac{E_{\max}}{c} = 6.14\times 10^{-8}T$$

Momentum and Radiation Pressure



$$p = \frac{T_{ER}}{c}$$ Momentum for complete absorption

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$ Pressure on surface

$$P = \frac{1}{A}\frac{d}{dt}\left(\frac{T_{ER}}{c}\right) = \frac{1}{c}\frac{\left(dT_{ER}/dt\right)}{A}$$

$$P = \frac{S}{c}$$

For complete reflection: $$p = \frac{2T_{ER}}{c}$$ $$P = \frac{2S}{c}$$

Pressure From a Laser Pointer

$\mathcal{P}_{laser}$ **= 3 mW, 70% reflection, *d* = 2 mm**

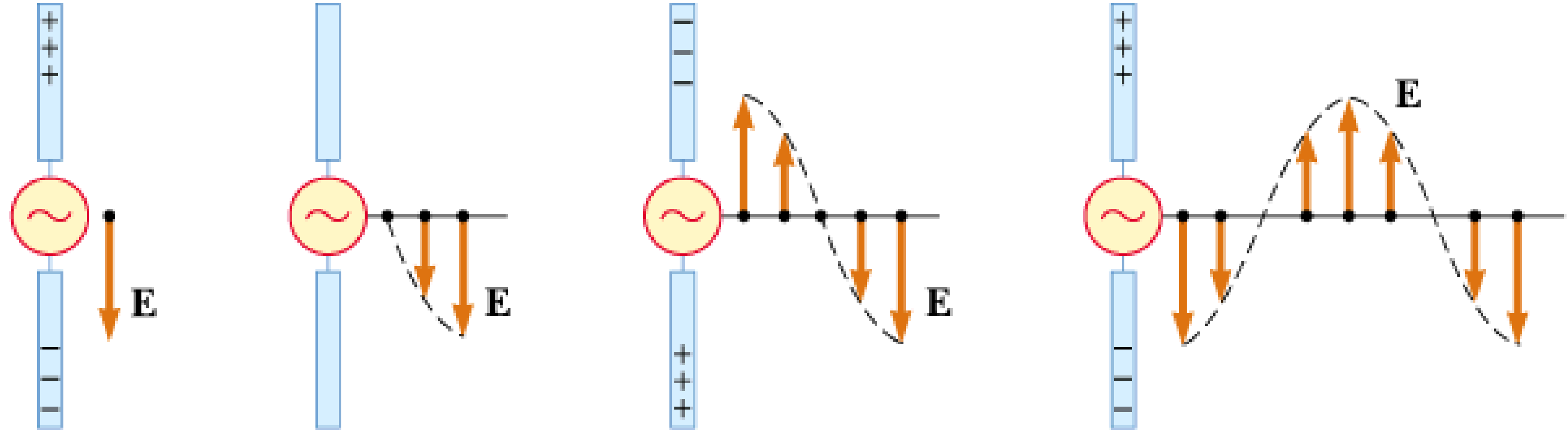
$$S = \frac{\mathcal{P}}{A} = \frac{0.003}{\left(\pi 0.001^2\right)} = 955 W/m^2$$

$$P = \frac{S}{c} + 0.7\frac{S}{c} = 1.7\frac{S}{c}$$

$$P = 1.7\frac{955}{3\times10^8} = 5.4\times10^{-6} N/m^2$$

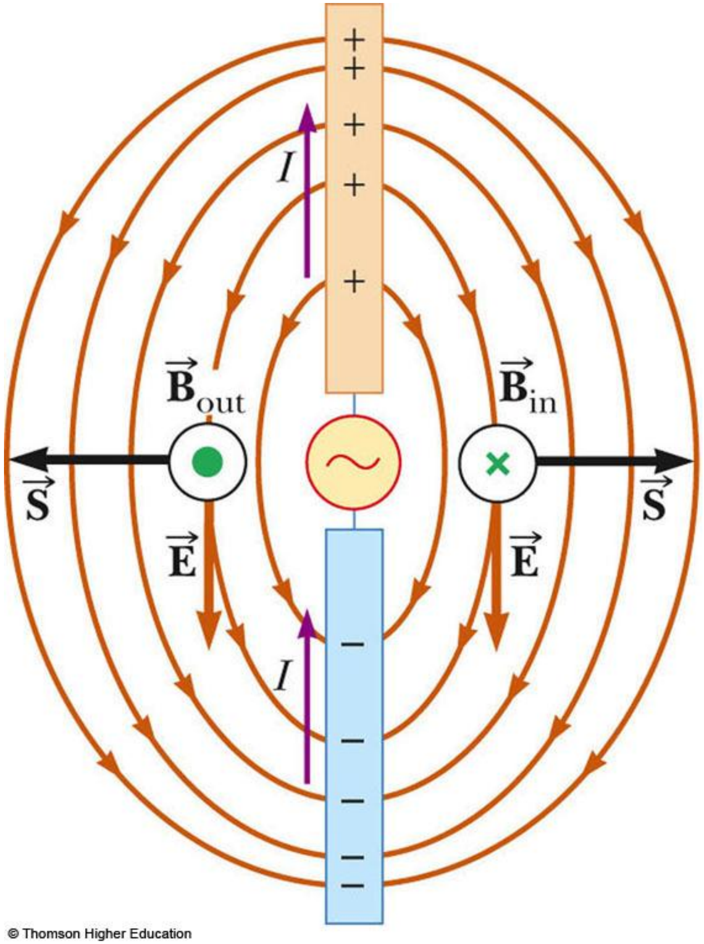
Antennas

The fundamental mechanism for electromagnetic radiation is an accelerating charge



(a) $t = 0$ (b) $t = \frac{T}{4}$ (c) $t = \frac{T}{2}$ (d) $t = T$

Half-Wave Antenna



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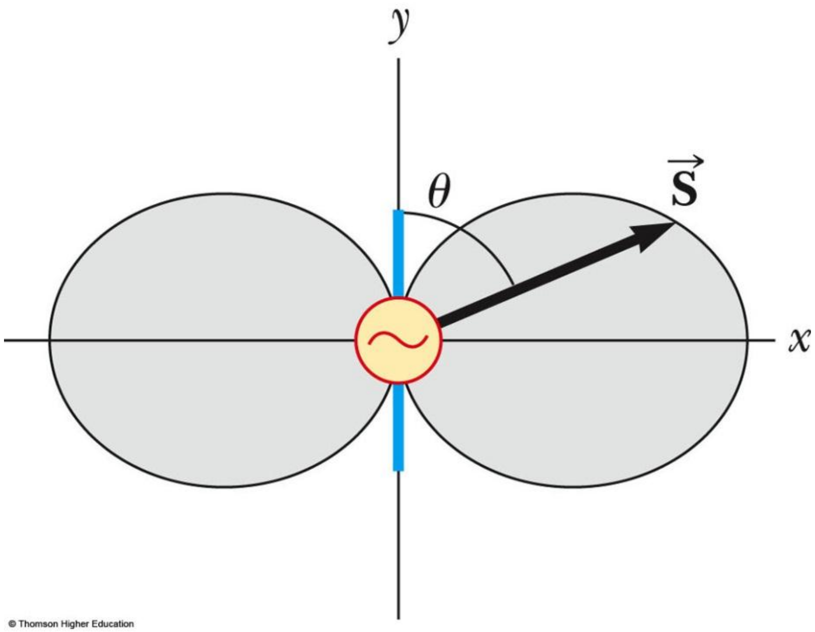

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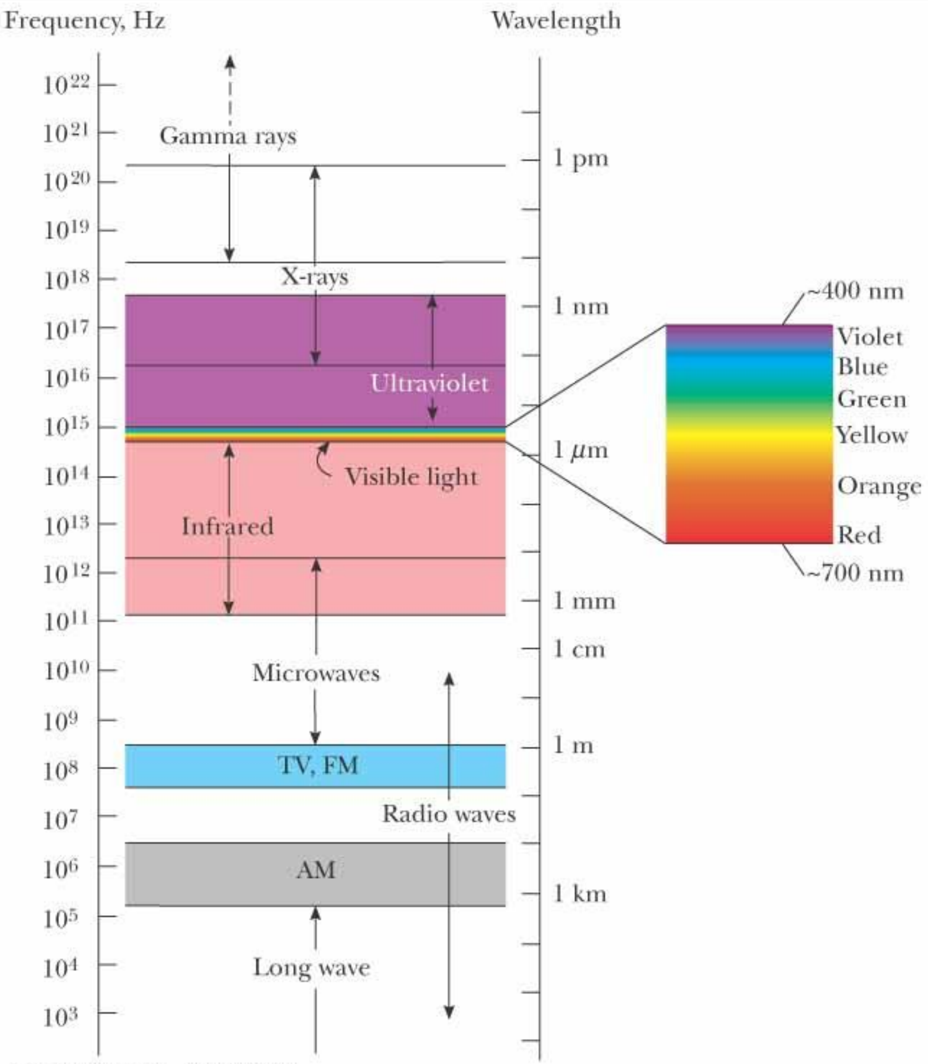
Electromagnetic Spectrum



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For Next Class

- Reading Assignment
  - Chapter 35: Nature of Light and Laws of Geometric Optics
- WebAssign: Assignment 12