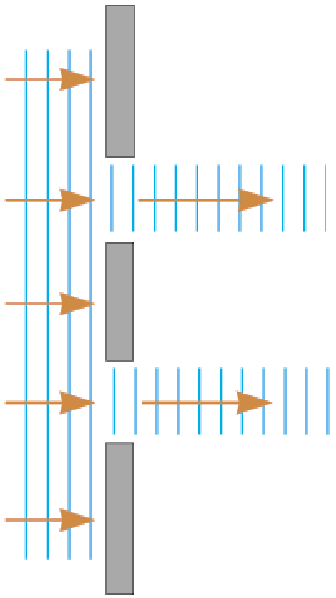
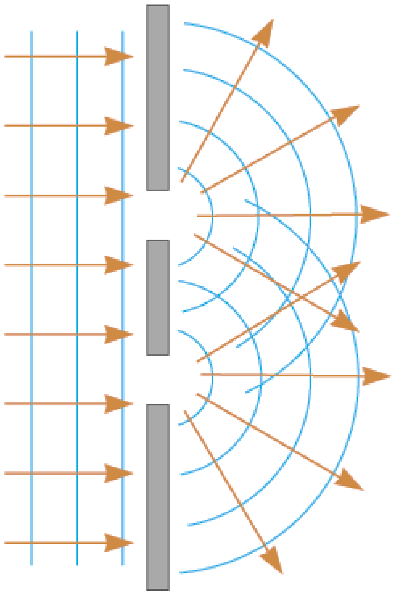
Diffraction and Polarization

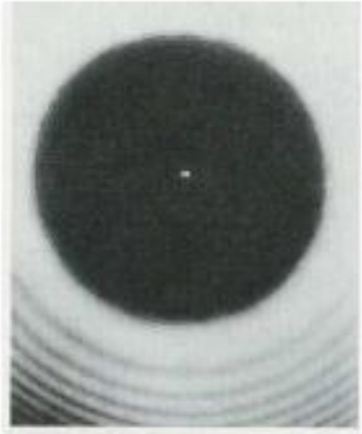
- Diffraction
- Diffraction from narrow slits
- Resolution issues
- The diffraction grating
- Polarization

Diffraction

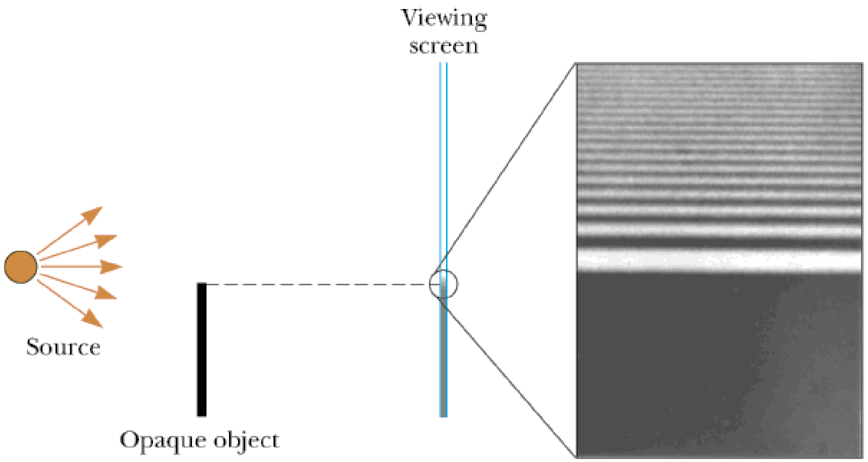




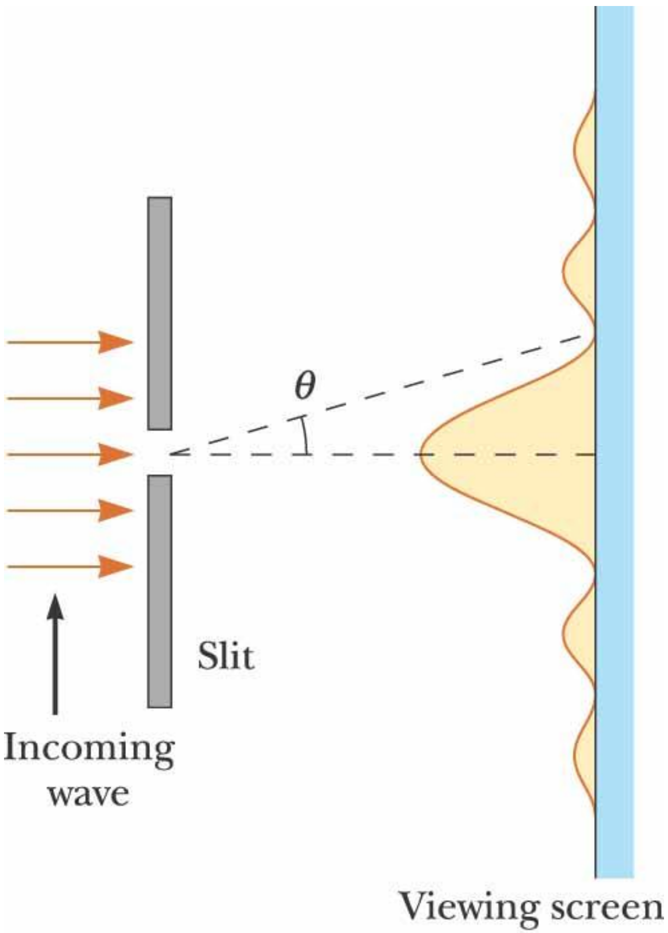
Light rays can bend around corners



No sharp boundary exists between the dark and bright parts on the screen



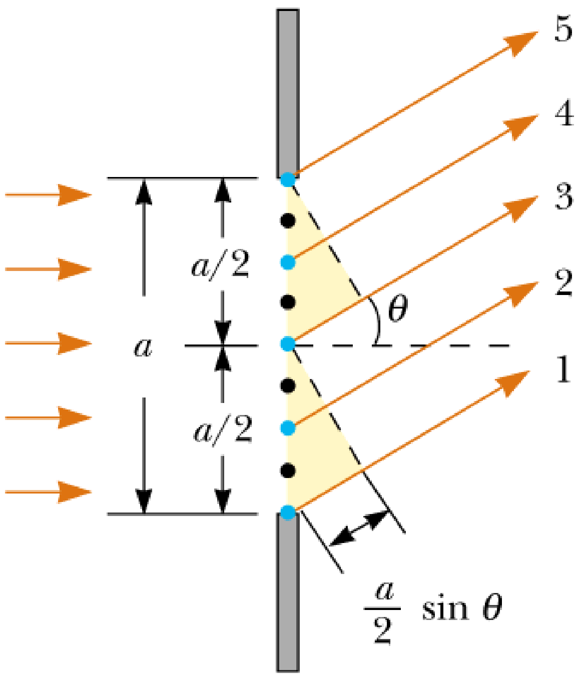
Fraunhofer (far field) Diffraction



The rays from the slit are parallel to each other.

Either the screen is far from the slit or a lens is used to focus the light rays from the slit.

Diffraction From Narrow Slits



Each portion of the slit acts as a wave source.

Divide slit into two parts $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$

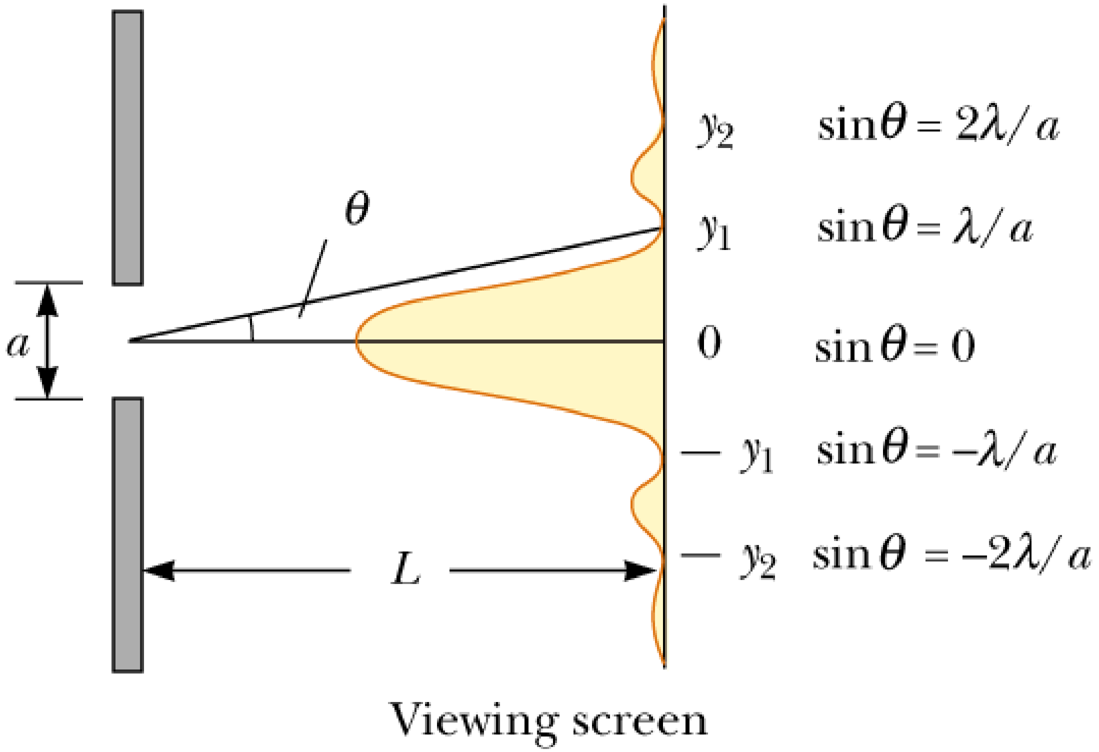
$$\sin\theta = \frac{\lambda}{a}$$

Now divide it into four parts $\sin\theta = \frac{2\lambda}{a}$

And into six parts $\sin\theta = \frac{3\lambda}{a}$

… so in general for destructive interference $\sin\theta = \frac{m\lambda}{a}$ $m = \pm1, \pm2, \pm3, \ldots$

Positions of Dark Fringes



$$\sin\theta_m = \frac{m\lambda}{a}$$

$$\tan\theta_m = \frac{y_m}{L}$$

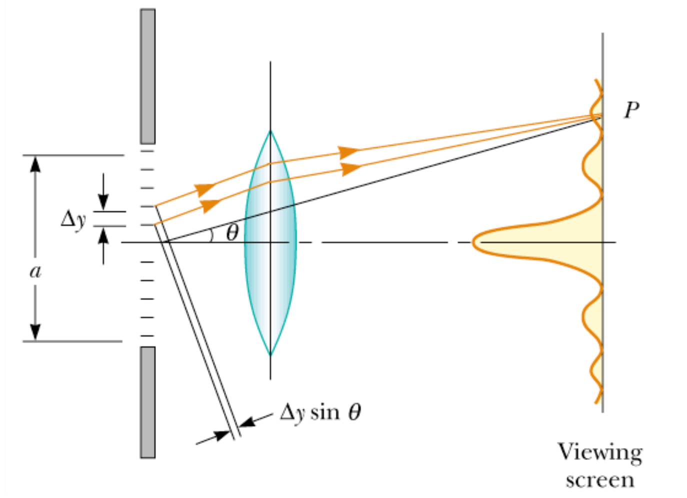
$$\tan\theta \approx \sin\theta$$

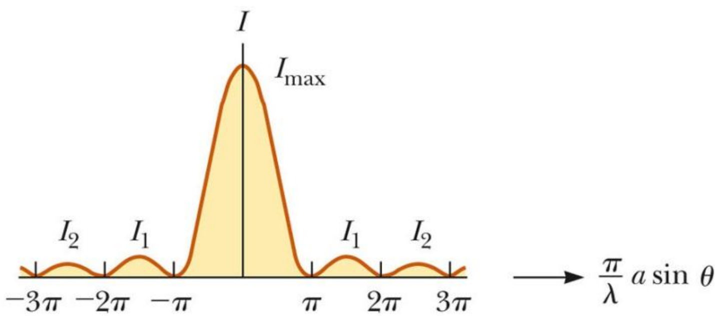
$$\frac{m\lambda}{a} \approx \frac{y_m}{L}$$

$$y_m \approx \frac{L}{a}\lambda m$$

$$m = \pm 1, \pm 2, \pm 3, \ldots$$

Intensity of the Single Slit Diffraction Pattern





More accurately, the pattern is a result of the interference of the waves coming from different parts of the slit.

Each Δy contributes an electric field of ΔE for the total field at P.

When adding the fields, we should take into account the relative phase differences of the fields.

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

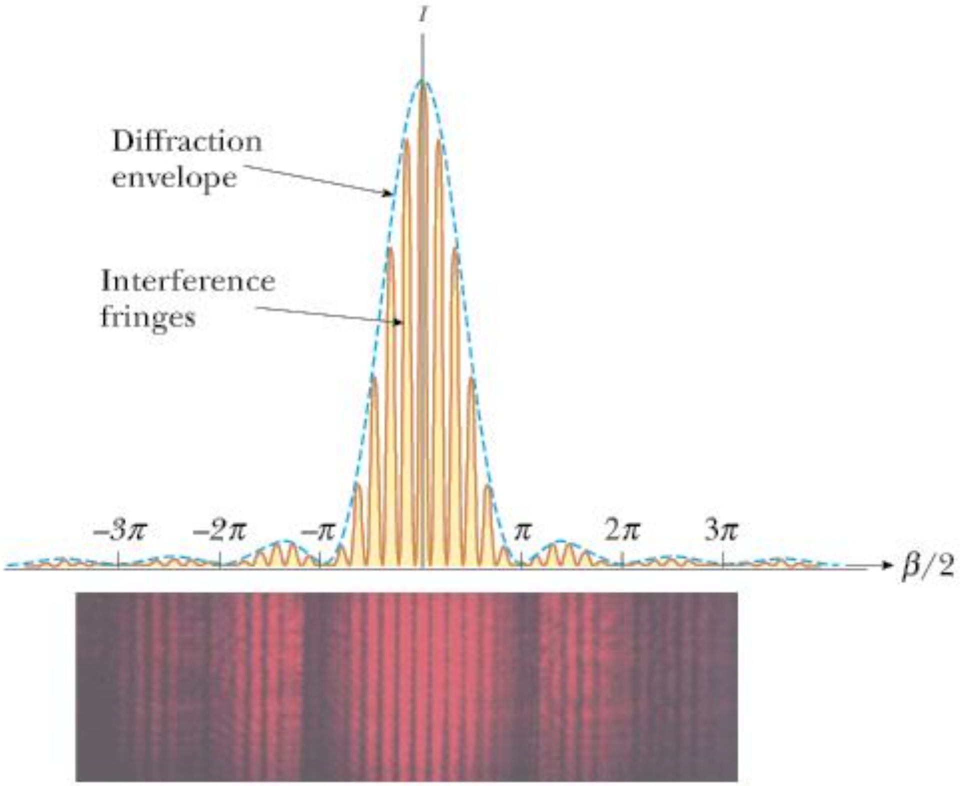
Adding up all these fields results in:

$$I = I_{max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

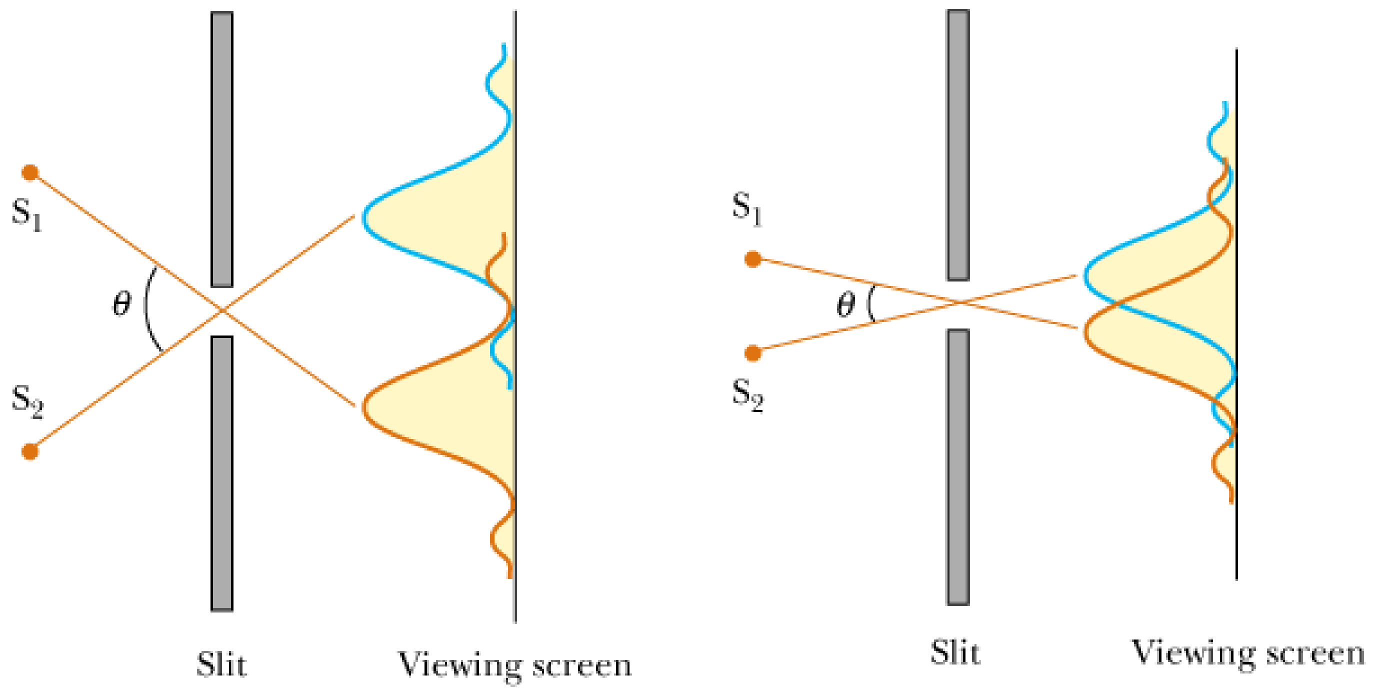
Two-Slit Diffraction Pattern

Interference and diffraction occur simultaneously

$$I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

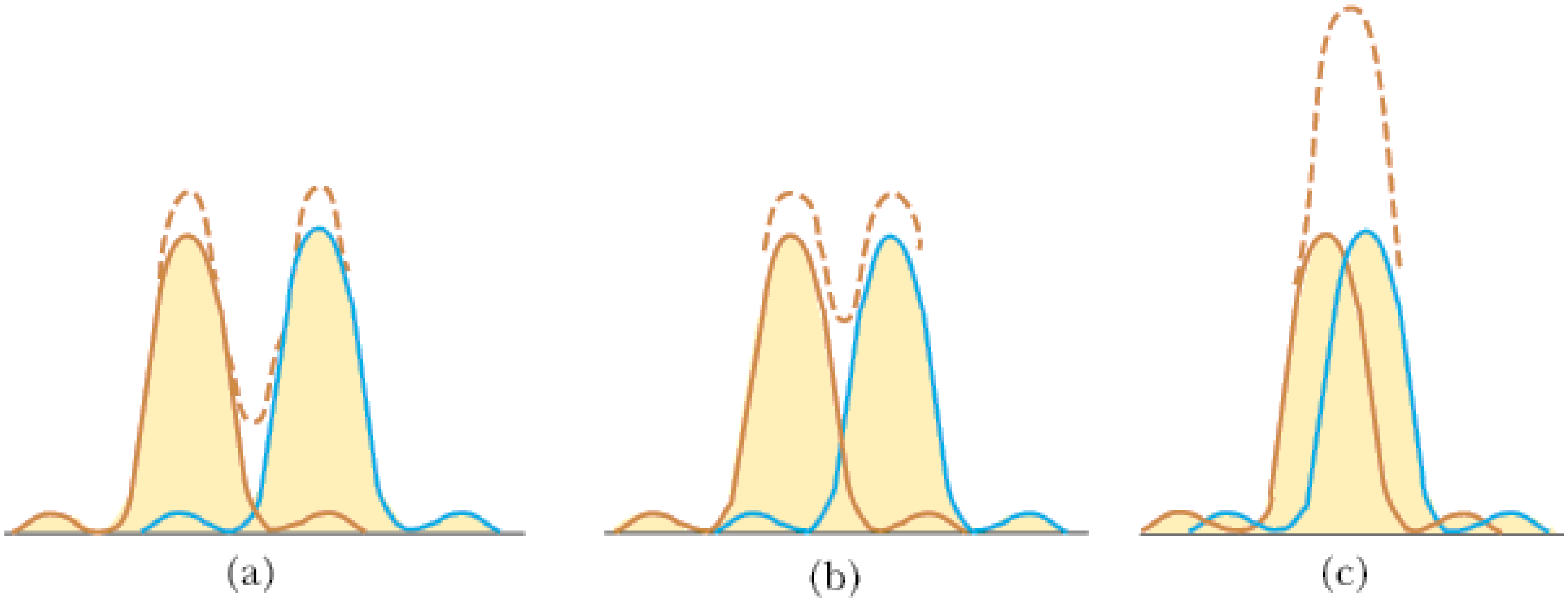


Resolution of Single Slit and Circular Apertures



When the central maximum of one image falls on the first minimum of the other Image, the images are said to be just resolved. This limiting condition is called the **Rayleigh's Criterion**.

Rayleigh's Criterion



$$\sin\theta = \frac{\lambda}{a} \longrightarrow \theta_{\min} = \frac{\lambda}{a}$$

For circular apertures $\theta_{\min} = 1.22\frac{\lambda}{D}$

Concept Question

For a given lens diameter, which light gives the best resolution in a microscope?

1. red
2. yellow
3. green
4. blue
5. All give the same resolution.

Limiting Resolution of a Microscope

$$\lambda = 589nm$$

$$D = 0.9cm$$

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{589\times10^{-9}}{0.9\times10^{-2}} = 7.98\times10^{-5}rad$$

$$\lambda = 400nm$$

$$\theta_{\min} = 1.22\frac{400\times10^{-9}}{0.9\times10^{-2}} = 5.42\times10^{-5}rad$$

$$n_{water} = 1.33$$

$$\lambda_{water} = \frac{\lambda_{vacuum}}{n_{water}} = \frac{589nm}{1.33} = 443nm$$

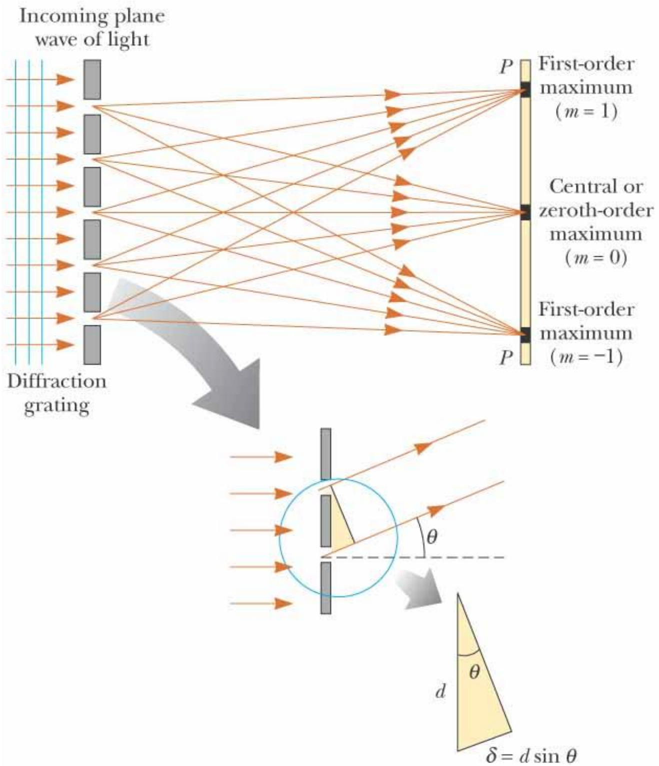
$$\theta_{\min} = 1.22\frac{443\times10^{-9}}{0.9\times10^{-2}} = 6\times10^{-5}rad$$

Spectroscopy

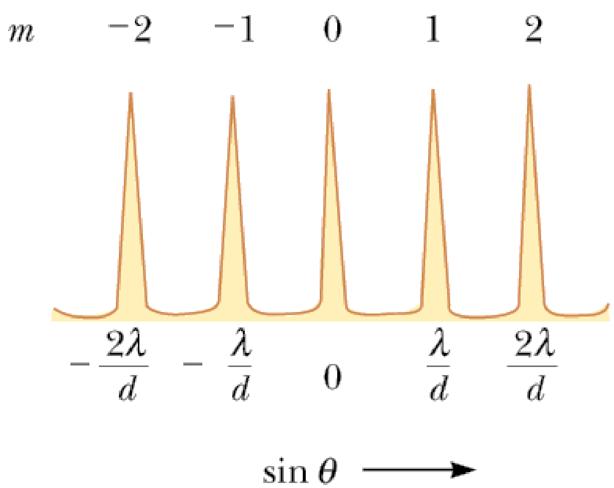
- For many experiments we would like to analyze the variation of light intensity as a function of wavelength.
- This light can be from a fluorescent source or light transmitted through a sample.
- A spectrometer will disperse light according to its wavelength and detect each wavelength component separately.

Diffraction Gratings

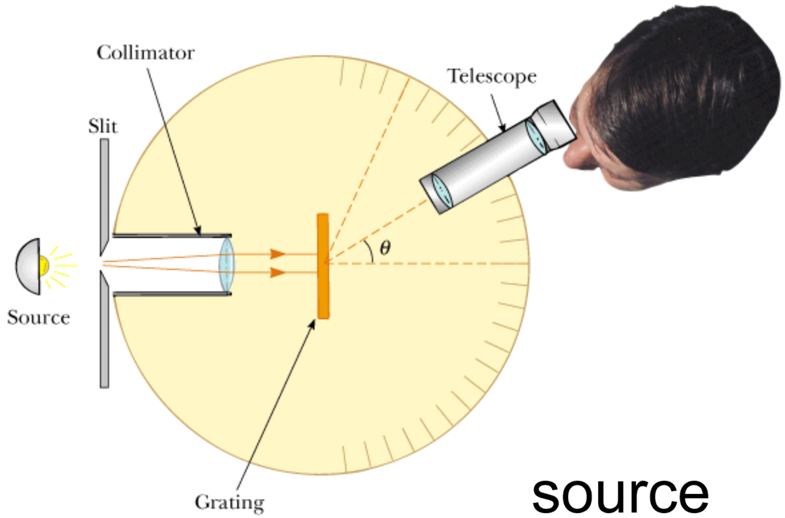
- A device with closely spaced slits (or grooves).
- Light passing through the slits will interfere with each other to produce a multiple slit interference pattern
- The position of the constructive peaks will be dependent on wavelength.

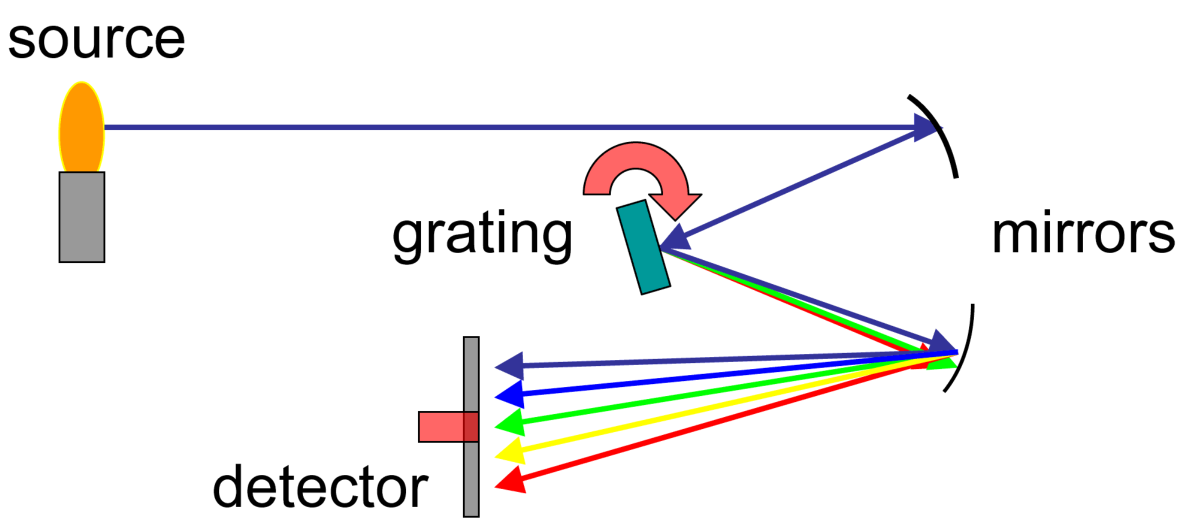


$$d \sin \theta = m\lambda$$



Spectrometers



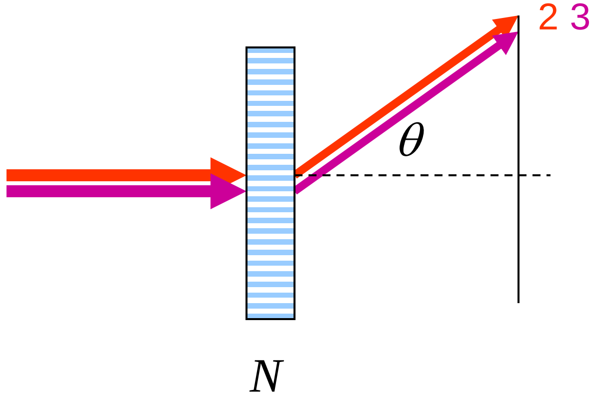


Concept Question

A diffraction grating is illuminated with yellow light at normal incidence. The pattern seen on a screen behind the grating consists of three yellow spots, one at zero degrees (straight through) and one each at ±45°.You now add red light of equal intensity, coming in the same direction as the yellow light. The new pattern consists of

1. red spots at 0° and ±45°.
2. yellow spots at 0° and ±45°.
3. orange spots at 0° and ±45°.
4. an orange spot at 0°, yellow spots at ±45°, and red spots slightly farther out.
5. an orange spot at 0°, yellow spots at ±45°, and red spots slightly closer in.

Problem 38.32



$$d = 1/N$$

$$\lambda_v = 400nm$$

$$\lambda_r = 600nm$$

$$\sin\theta = \frac{m\lambda}{d}$$

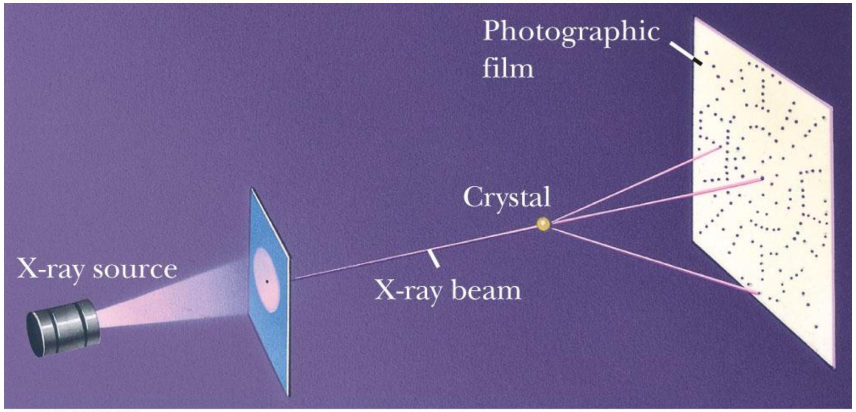
$$\sin\theta_{v3} = \frac{3(400nm)}{d} = 3N(400nm)$$

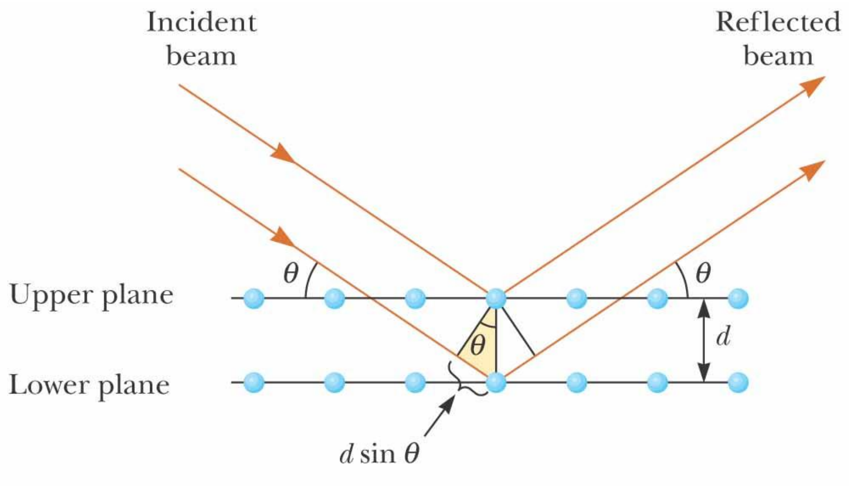
$$\sin\theta_{r2} = \frac{2(600nm)}{d} = 2N(600nm)$$

$$\sin\theta_{v3} = \sin\theta_{r2}$$

$$\theta_{v3} = \theta_{r2}$$

X-Ray Diffraction



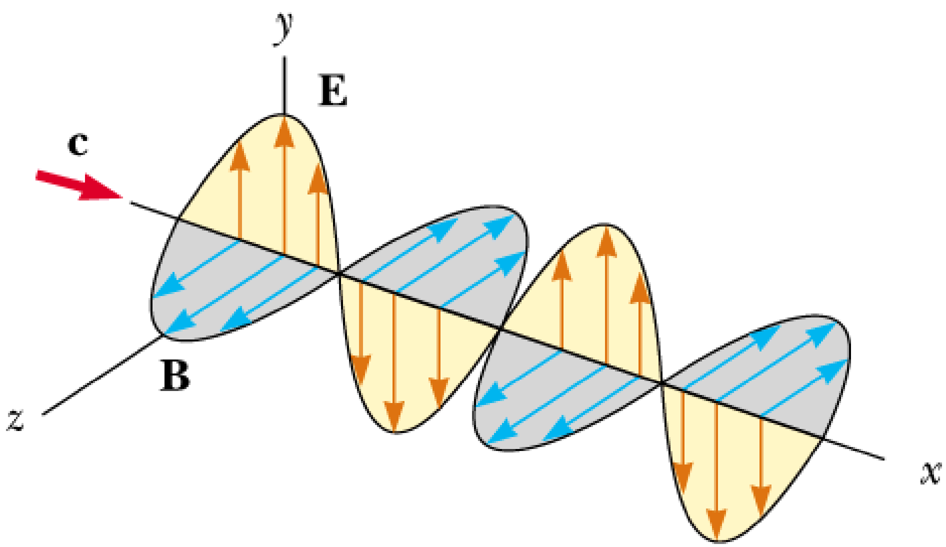


- An X-ray has a wavelength on the order of the atomic separation in crystals (~0.1 nm).
- Successive layers of atoms act as a diffraction grating for the X-ray and create an interference pattern.
- The pattern is a characteristic of the crystal structure.

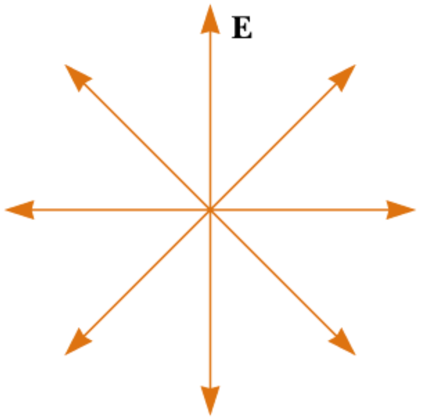
$$2d\sin\theta = m\lambda$$

$$m = 1,2,3,\ldots$$

Polarization of Light Waves



An EM wave **linearly** polarized in the y direction

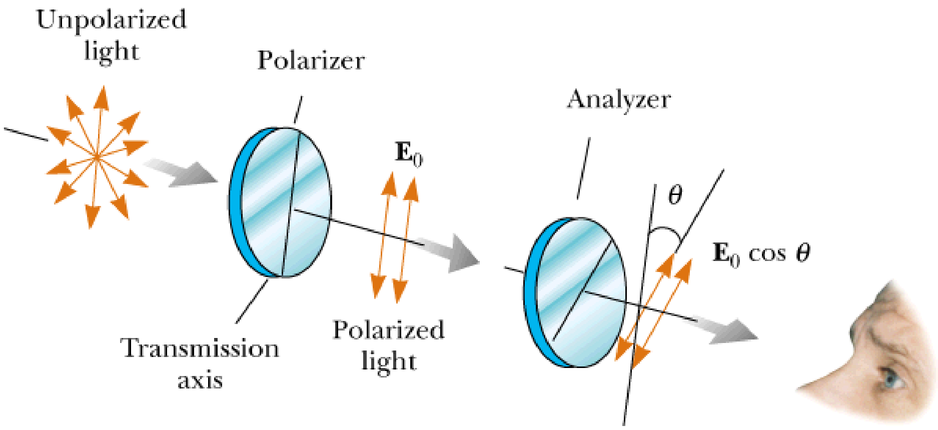


An unpolarized field

A linearly polarized field

Polarization by Selective Absorption

- These polarizers work by transmitting light along their transmission axis.
- The transmitted light will have a polarization along the transmission axis.
- Unpolarized light will be transmitted as a linearly polarized wave at half the intensity.

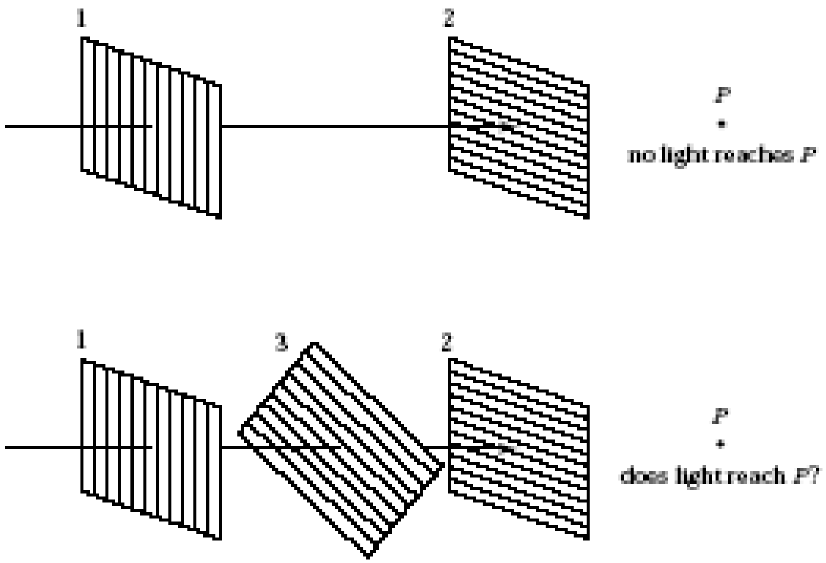


$$I = I_{max} \cos^2 \theta$$

Malus' Law

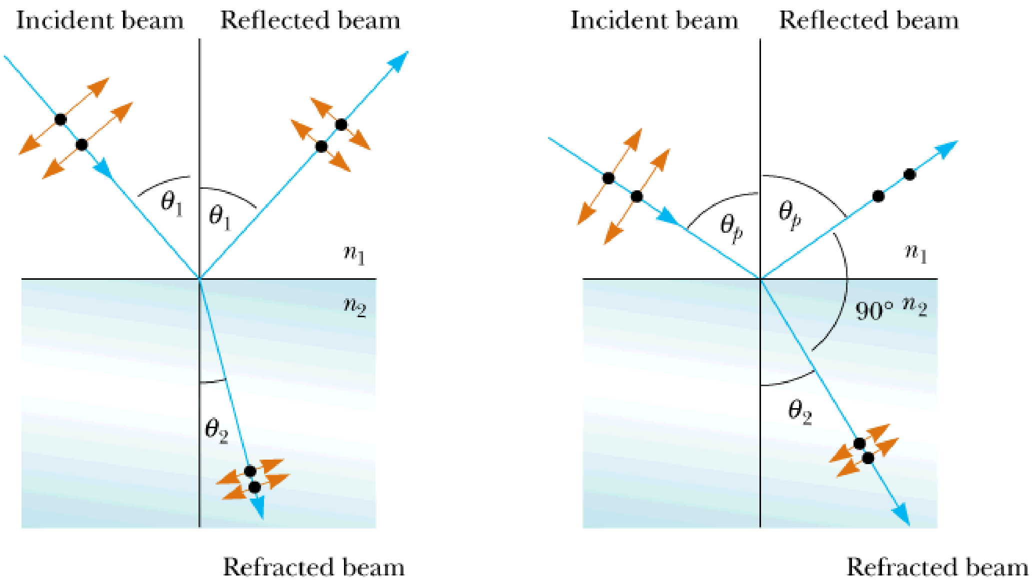
Concept Question

When a ray of light is incident on two polarizers with their polarization axes perpendicular, no light is transmitted. If a third polarizer is inserted between these two with its polarization axis at 45° to that of the other two, does any light get through to point *P*?



1. yes
2. no

Polarization by Reflection



The polarization of reflected and refracted rays are dependent on the angle of incidence. For one angle, the reflected light is completely linearly polarized.

$$\theta_p + \theta_2 = 90°$$

$$\sin\theta_2 = \sin(90 - \theta_p) = \cos\theta_p$$

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\cos\theta_p}$$

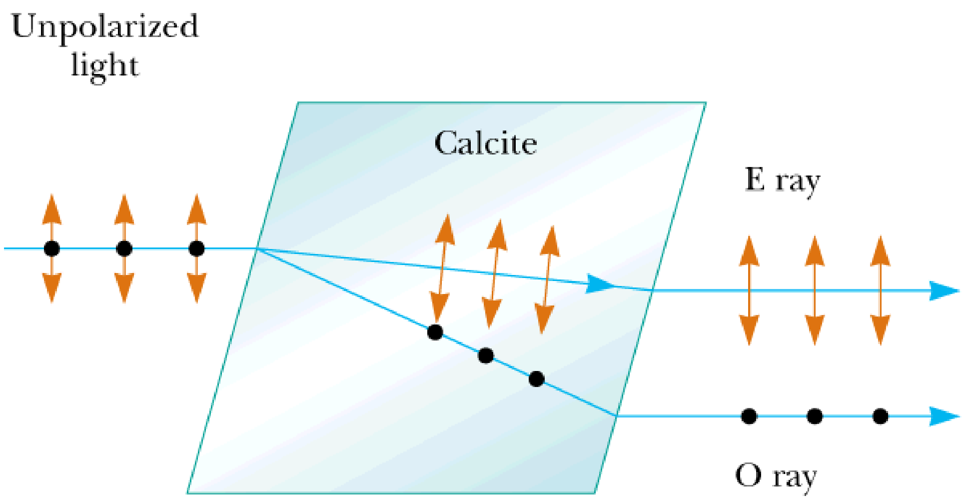
Brewster's Angle

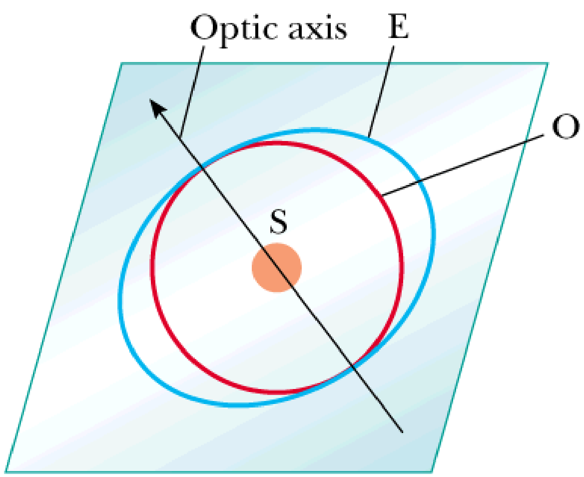
$$n = \tan\theta_p$$

Polarization by Double Refraction

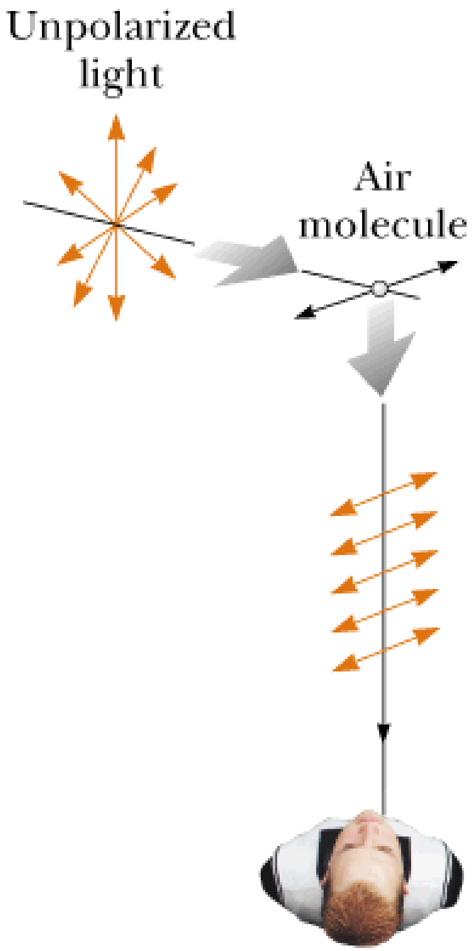
In an amorphous material: n is the same for all polarizations
In some crystalline materials: n depends on the direction of polarization

Birefringent materials: ordinary (n_o) and extra-ordinary (n_e) indices of refraction





Polarization by Scattering



Examples:
Blue skies, red sunsets

Optical Activity

- Optically active materials are materials that rotate the polarization of any light transmitted through them.
- This is due to molecular asymmetry which can be caused by the molecular structure or external factors such as mechanical stress.

For Next Class

- Review for the Final on Monday
- WebAssign: Assignment 16, due Monday 11 am
- Final on Tuesday, 9:00 am