Read Section 1.1

Lecture 1: Functions

Definition - A function f is a rule that assigns to each x in a set D exactly one element, called $f(x)$, in a set E.

* D is the domain.
* E is the range.
* x is the independent variable
* f is the dependent variable.

example (formula):

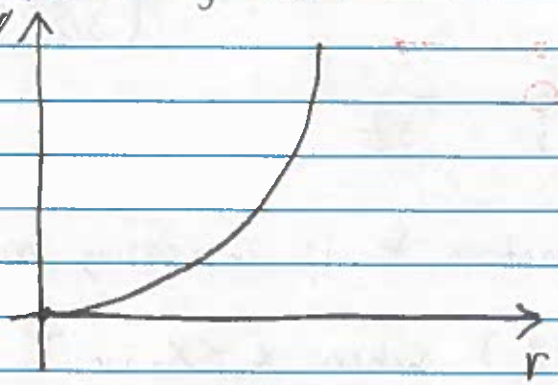
Volume of a sphere of radius r:

$$V(r) = \frac{4}{3}\pi r^3$$

Domain: $r \geq 0$, $\{r : r \geq 0\}$ → set builder notation

Range: $V \geq 0$, $\{V : V \geq 0\}$

"the set of all r such that $r \geq 0$"



The graph of a function is the set of ordered pairs:

$$\{(r, V(r)) : r \in D\}$$

example (table):

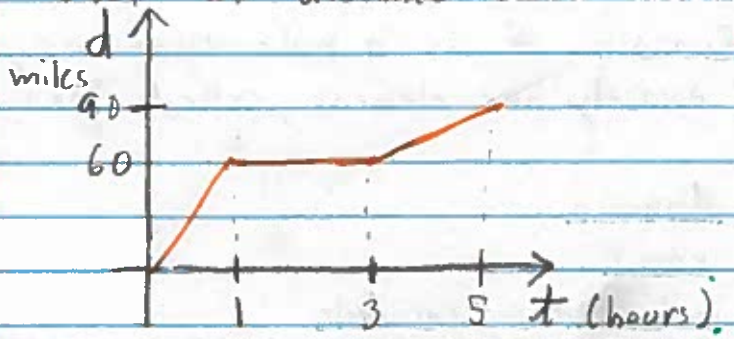
School Year	# students
1	1285
2	1127
3	1118
4	1113

$D = \{1, 2, 3, 4\}$

$E = \{1285, 1127, 1118, 1133\}$

example:

Plot of distance $d(t)$ travelled by a car:

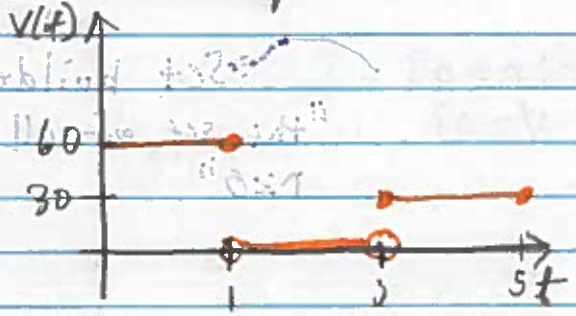


$$d(t) = \begin{cases} 60t, & 0 \le t \le 1 \\ 60, & 1 < t < 3 \\ 30t - 30, & 3 \le t \le 5 \end{cases}$$

Domain $= \{t : 0 \le t \le 5\}$

Range $= \{d : 0 \le d \le 90\}$.

Sketch a graph of the velocity $v(t)$



$$V(t) = \begin{cases} 60, & 0 \le t \le 1 \\ 0, & 0 < t < 3 \\ 30, & 3 \le t \le 5 \end{cases}$$

Definition – A function f is increasing on an interval I if

$f(x_1) < f(x_2)$ when $x_1 < x_2$ in I.

It is called decreasing on I if

$f(x_1) > f(x_2)$ when $x_1 < x_2$ in I.

* In the example above d is increasing on $[0,1]$ and $[3,5]$.

example:

If $f(x) = ax^2+bx+c$, what is

$$\frac{f(x+h)-f(x)}{h} = \frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}$$

$$= \frac{ax^2+2axh+ah^2+bx+bh-ax^2-bx-c}{h}$$

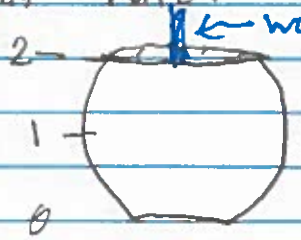
$$= \frac{2axh+ah^2+bh}{h}$$

$$= \frac{h(2ax+ah+b)}{h}$$

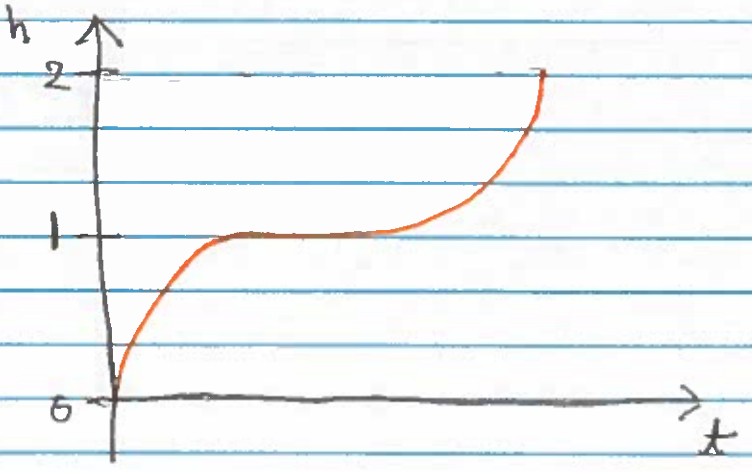
$$= 2ax+ah+b$$

example:

Suppose water fills the following barrel at a constant rate:



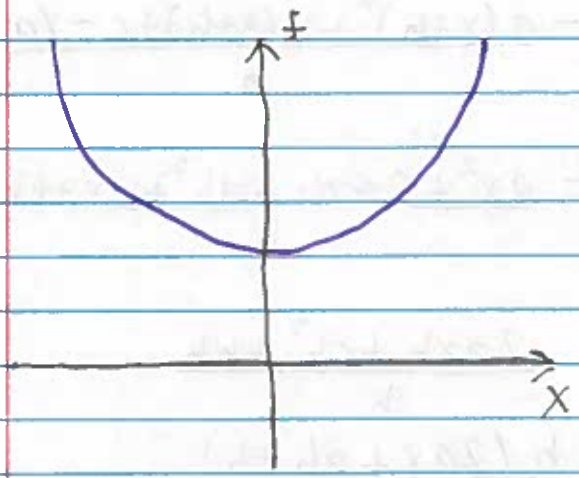
Sketch a graph of the height of the water as a function of time.



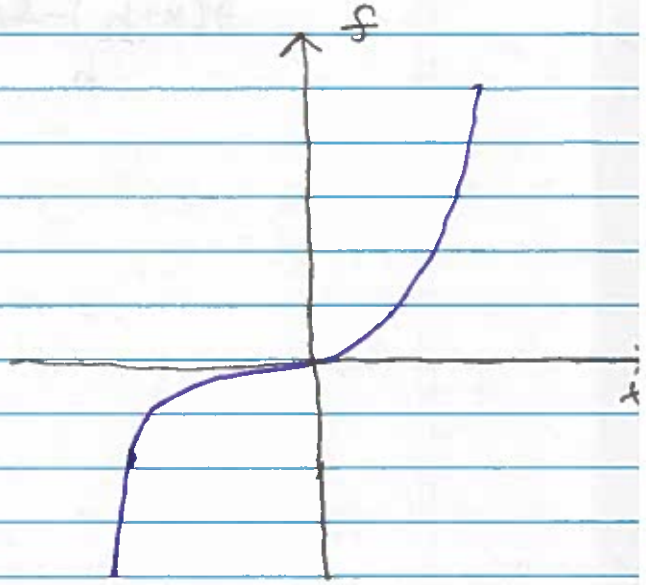
Symmetry:

A function is odd if $f(x) = -f(-x)$.

A function is even if $f(x) = f(-x)$



even function



odd function

Example:

Which functions are odd or even:

- $f(x) = 3x^2 + 2$
- $f(x) = 4x^3 + 3x$
- $f(x) = x^3 + 1$.