Lecture #2: Essential Functions.

Polynomials.

A function P is a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

a_i are coefficients

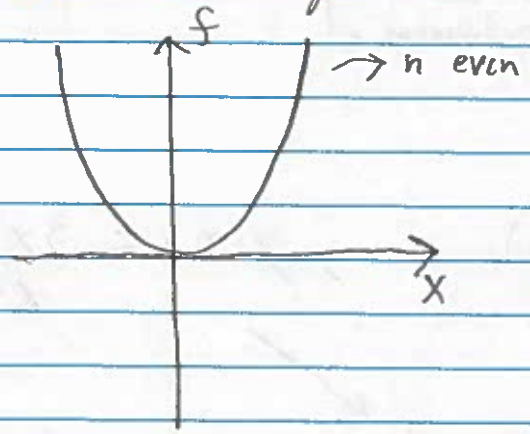
n is the degree

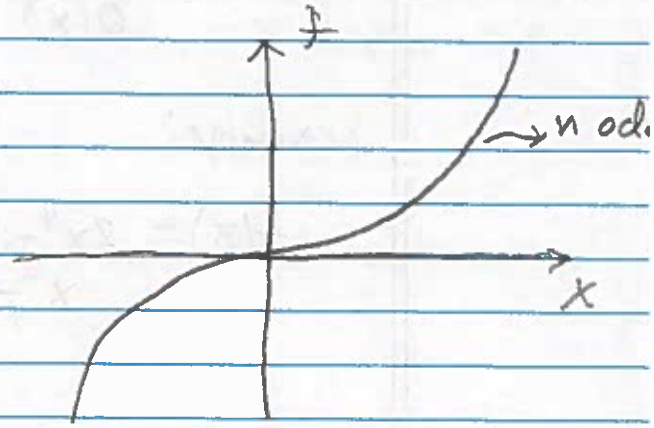
Fundamental Theorem of Algebra - An n-th degree polynomial has at most n real roots and imaginary roots come in pairs.

Power Functions.

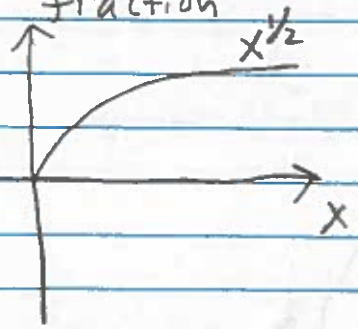
$$f(x) = x^n$$

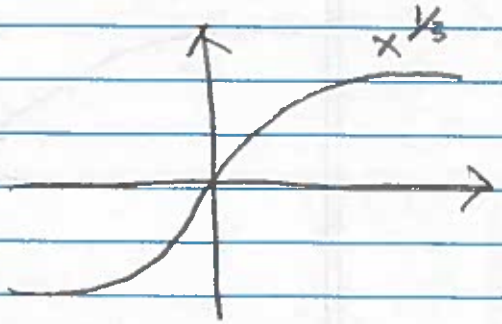
- n is an integer:





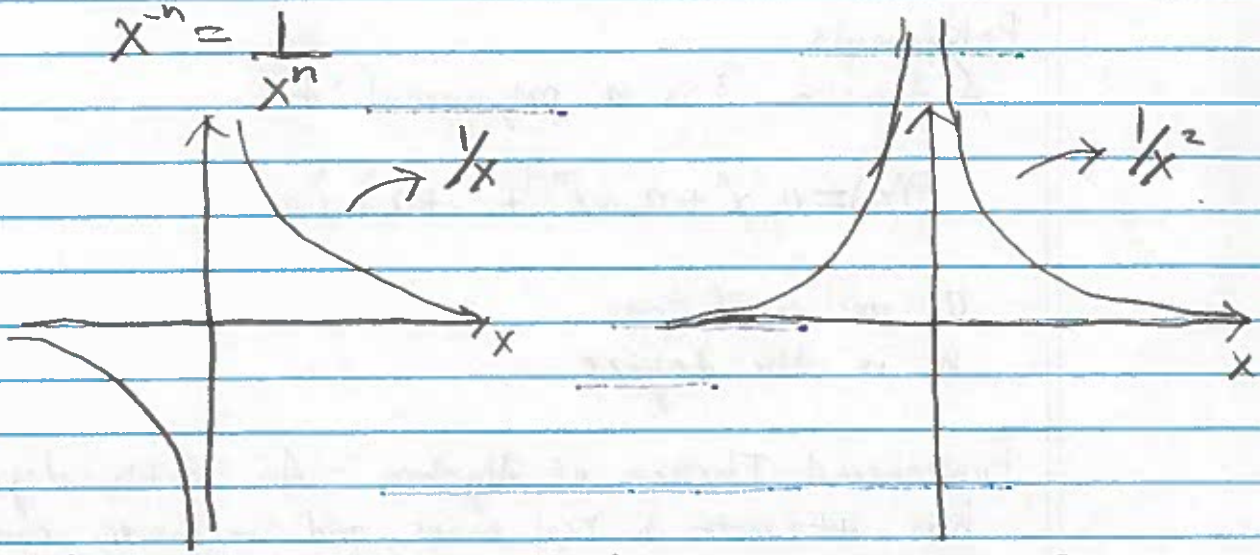
- n a fraction





- n < 0 is negative

$$x^{-n} = \frac{1}{x^n}$$



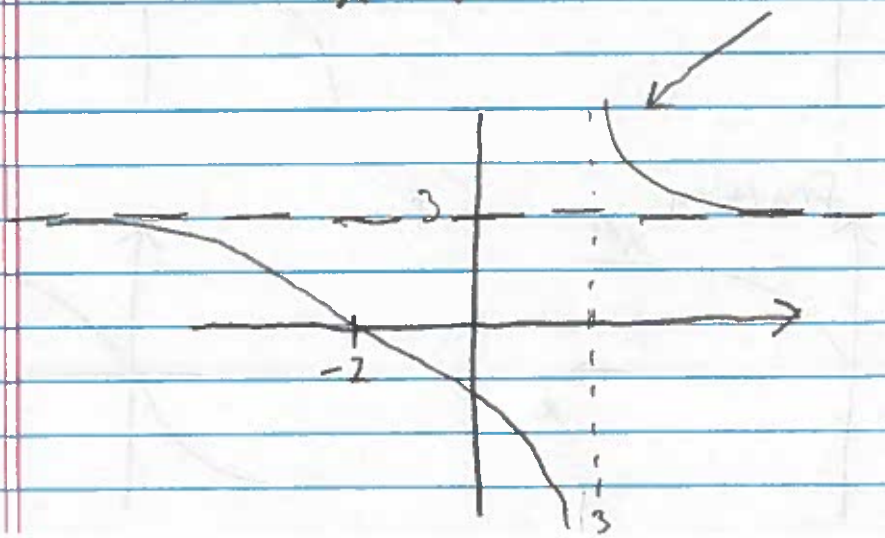
What is the domain and range of these functions. What are the limits.

Rational Functions.

$$f(x) = \frac{P(x)}{Q(x)}$$ - polynomial / polynomial

example:

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}, \quad g(x) = \frac{3x+2}{x-3}$$



Trig Functions

* $\sin(x)$, $\cos(x)$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$, $\csc(x) = \frac{1}{\sin(x)}$

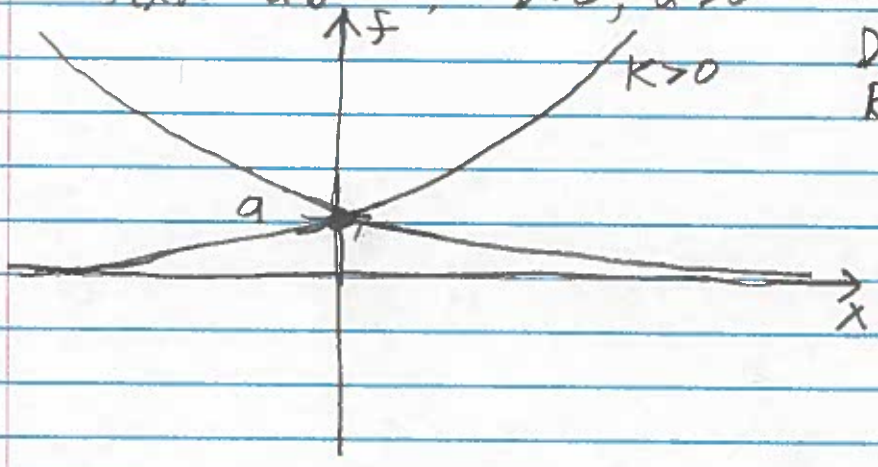
$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$

* $\sin^2(x) + \cos^2(x) = 1$
* $-1 \le \sin(x) \le 1$, $-1 \le \cos(x) \le 1$

* $\sin(x + 2\pi) = \sin(x)$

Exponential Functions:

$f(x) = ab^{kx}$, $b > 0$, $a > 0$

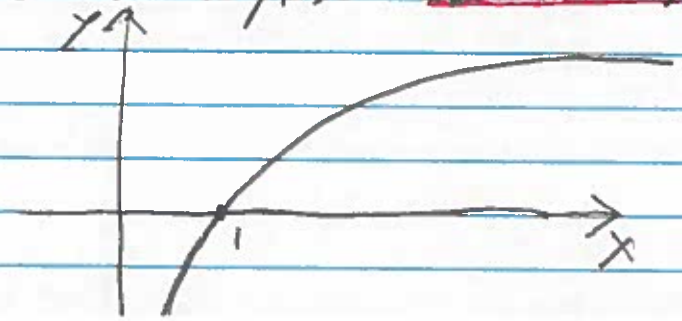


Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Logarithmic Functions

If $f(x) = b^x$, what is $f^{-1}(x)$?

$f^{-1}(x) = \log_b x$ (Definition)



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$