

Homework 3

Numerical Linear Algebra

September 25, 2017

1 Problems for everybody

1. Problems 6.1, 6.4, 7.1
2. Recall, for any subspace $V \subset \mathbb{R}^n$ that

$$V^\perp = \{\vec{v} \in \mathbb{R}^n : \forall \vec{w} \in V, \vec{v}^T \vec{w} = 0\}.$$

Let $A \in \mathbb{R}^{n \times n}$. Prove the following:

- (a) $\text{null}(A^T) = (\text{range}(A))^\perp$,
- (b) $\text{range}(A^T) = (\text{null}(A))^\perp$,
- (c) $\text{null}(A) = (\text{range}(A^T))^\perp$,
- (d) $\text{range}(A) = (\text{null}(A^T))^\perp$.

2 Problem for math undergraduate and education majors

1. Suppose we want to solve $A\vec{x} = \vec{b}$. What condition on \vec{b} must be satisfied in order to guarantee the existence of a solution to this problem. **Hint:** Problem 2 above is an incredibly useful theorem that might be relevant to this problem.

3 Problem for math graduate students

1. Let $A, B \in \mathbb{R}^n$ and suppose $C = A + B$. Let $\sigma_a, \sigma_b, \sigma_c$ be the largest singular values of A, B , and C respectively. Form a conjecture about how σ_c relates to σ_a and σ_b . Prove this conjecture.
2. Problem 6.5