Homework 6

Numerical Linear Algebra

October 4, 2017

1 Problems for everybody

1. Let
$$\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \text{ and } H = (\text{span}\{\vec{v}\})^{\perp}.$$
   - Find the matrix $P_{\vec{v}}$, the orthogonal projection onto $\text{span}\{\vec{v}\}$.
   - Find the matrix P_H, the orthogonal projection onto H.
   - Find Q_H, the unitary matrix that reflects across H.
2. Let A be an $m \times n$ matrix ($m \geq n$) and let $A = \hat{Q}\hat{R}$ be a reduced QR factorization of A. Prove that A has full rank if and only if the diagonal entries of R are nonzero.
3. Create your own QR algorithm in Matlab based off of the Gram-Schmidt process. The code should take in an $m \times n$ matrix and output the reduced matrices $\hat{Q}$ and $\hat{R}$. Submit a printed out copy of your code. **Be sure to test your code!**
4. Let Z be the following matrix:
$$Z = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}.$$
   - Use your code from part 2 to compute the reduced QR factorization of this matrix.
   - Use Matlab's built in algorithm $QR(Z, 0)$ to compute the reduced QR factorization of Z.
   - Compare the outputs of these two algorithms and comment on any differences you see.
5. Problem 10.2.

2 Problems for MST graduate students

1. Problems 7.3, 11.1.