Homework 7

Numerical Linear Algebra

October 13, 2017

1 Problems for everybody

1. Let $\vec{x}, \vec{y} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$. Find asymptotic formulas for the number of floating point operations (flops) for the following calculations:
   - $\vec{x}^* \vec{y}$
   - $A\vec{x}$
   - AB
2. Create a Matlab function that takes in a matrix $A \in \mathbb{R}^{n \times n}$ and outputs the inverse of A. Your code needs to use the QR algorithm as well as back substitution to find the inverse of A. **Hint:** To form the columns of A^{-1} remember that $A^{-1}e_j$ will give you the $j-th$ column of A^{-1}. You can use this to set up an equation for the $j-th$ column of A^{-1} which can be solved using the QR algorithm and back substitution.
3. Problem 12.3.
4. Consider the following Matlab code provided below. Show that this algorithm satisfies:

$$\text{flops} \sim \frac{2}{3}n^3$$

```
function [U,L] = HW7Prob1(A)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   This code does something to a matrix.
%
%   Inputs:
%
%   1. A an nxn matrix.
%
%   Outputs:
%   1. L a matrix
%   2. U a matrix
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[n,¬]=size(A);

U=A;
L=eye(n);

for k=1:n-1,
    for j=k+1:n,
        L(j,k)=U(j,k)/U(k,k);
        U(j,k:n)=U(j,k:n)-L(j,k)*U(k,k:n);
    end
end
```

5. Consider the following Matlab code provided below. Show that this algorithm satisfies:

$$\text{flops} \sim \frac{1}{3}n^3$$

```
function [R] = HW7Prob2(A)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   This code does something to a matrix.
%
%   Inputs:
%
%   1. A an nxn symmetric positive definite matrix.
%
%   Outputs:
%   1. R a matrix
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[n,¬]=size(A);
R=A;

for k=1:n,
    for j=k+1:n,
        R(j,j:n)=R(j,j:n)-R(k,j:n)*R(k,j)/R(k,k);
    end
    R(k,k:n)=R(k,k:n)/sqrt(R(k,k));
end
```

6. Consider the following Matlab code provided below. Show that this algorithm satisfies:

$$\text{flops} \sim \frac{10}{3}n^3$$

```
function [H] = HW7Prob3(A)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   This code does something to a matrix.
%
%   Inputs:
%
%   1. A an nxn matrix.
%
%   Outputs:
%   1. H a matrix
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[n,¬]=size(A);
H=A;

for k=1:n-2,
    x=H(k+1:n,k);
    v=sign(x(1))*norm(x)*eye(n-k,1)+x;
    v=v/norm(v);
    H(k+1:n,k:n)=H(k+1:n,k:n)-2*v*(v'*H(k+1:n,k:n));
    H(1:n,k+1:n)=H(1:n,k+1:n)-2*(H(1:n,k+1:n)*v)*v';
end
```