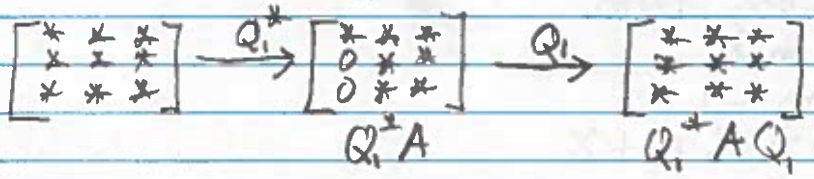
Hessenberg Form -

Schur Factorization

$$A = Q T Q^*$$

(unitary)

Householder to triangular?



Why is the first column filled in again?

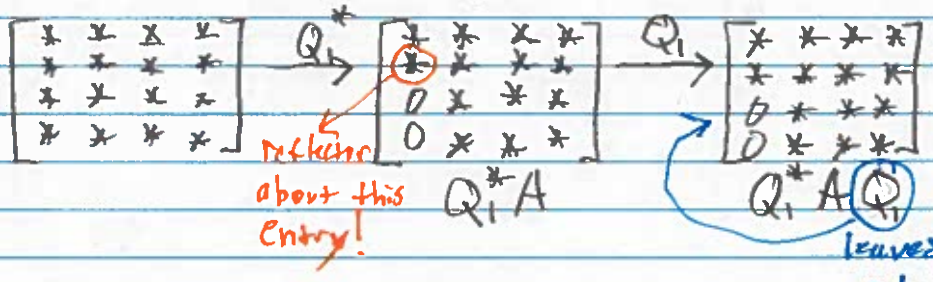
$$Q_1^* A Q_1 = Q_1^* (Q_1^* A^*)^*$$

↓

Does Householder reflection to columns!

Not possible to fix! (Thanks Abel ∴).

Householder to Hessenberg



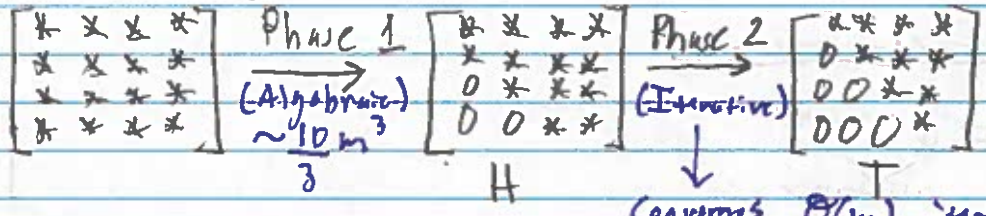
Final Product

$$\underbrace{Q_{n-2}^* \cdots Q_1^*}_{Q^*} A \underbrace{Q_1 \cdots Q_{n-3}}_{Q} = H = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{bmatrix}$$

$$\rightarrow A = Q H Q^*$$

same eigenvalues!

Eigenvalue Algorithms



Algorithm Hessenberg Form

For $k = 1$ to $m-2$

$x = A(k+1:m, k)$

$v_k = \text{sign}(x_1) \cdot \|x\|_2 e_1 + x$

$v_k = v_k / \|v_k\|_2$

$A_{k+1:m,\,k:m} = A_{k+1:m,\,k:m} - 2 v_k (v_k^* A_{k+1:m,\,k:m})$

$A_{1:m,\,k+1:m} = A_{1:m,\,k+1:m} - 2 (A_{1:m,\,k+1:m} v_k) v_k^*$

Work $\sim \frac{10}{3} m^3$.