

## Hessenberg Form -

### Schur Factorization

$$A = Q T Q^*$$

unitary

### Householder to triangular?

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \xrightarrow{Q_1^*} \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & * & * \end{bmatrix} \xrightarrow{Q_1} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$Q_1^* A$                        $Q_1^* A Q_1$

Why is the first column filled in again?

$$Q_1^* A Q_1 = Q_1^* (Q_1^* A^*)^*$$

Does Householder reflection to columns!

Not possible to fix! (Thanks Abel!)

### Householder to Hessenberg

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \xrightarrow{Q_1^*} \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} \xrightarrow{Q_1} \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix}$$

$Q_1^* A$                        $Q_1^* A Q_1$

reflection about this entry!

leaves first column unchanged!

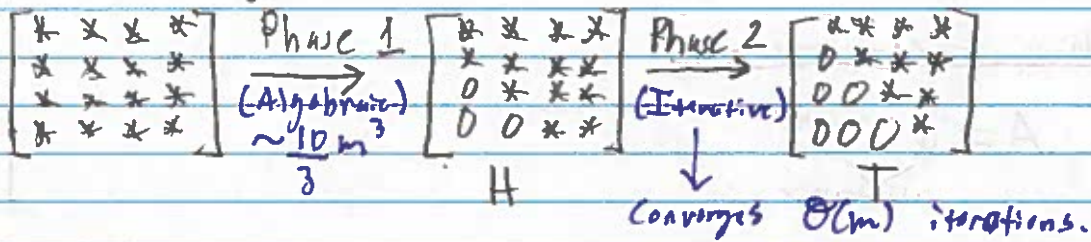
### Final Product

$$Q_{n-2}^* \dots Q_1^* A Q_1 \dots Q_{n-2} = H = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{bmatrix}$$

$$\rightarrow A = Q H Q^*$$

same eigenvalues!

# Eigenvalue Algorithms.



## Algorithm Hessenberg Form

For  $k=1$  to  $m-2$

$$x = A(k+1:m, k)$$

$$v_k = \text{sign}(x_1) \cdot \|x\|_2 e_1 + x$$

$$v_k = v_k / \|v_k\|_2$$

$$A_{k+1:m, k+1:m} = A_{k+1:m, k+1:m} - 2 v_k (v_k^* A_{k+1:m, k+1:m})$$

$$A_{1:m, k+1:m} = A_{1:m, k+1:m} - 2 (A_{1:m, k+1:m} v_k) v_k^*$$

Work  $\sim \frac{10}{3} m^3$ .