

MST 383/683

Homework #5

Due Date: October 29 2021

1. Malaria is a disease that can be transmitted to humans by mosquitoes and is also transmitted to mosquitoes by humans. A percentage of pregnant women who are infected with malaria give birth to malaria-infected newborns. The dynamics of the disease can be modeled as follows:

$$\begin{aligned}\dot{S}_m &= \mu_m(I_m + S_m) - \beta_m S_m I - \mu_m S_m, \\ \dot{I}_m &= \beta_m S_m I - \mu_m I_m, \\ \dot{S} &= \mu(S + \sigma I + R) - \beta S I_m - \mu S + \gamma R, \\ \dot{I} &= (1 - \sigma)\mu I + \beta S I_m - (\mu + \alpha)I, \\ \dot{R} &= \alpha I - (\mu + \gamma)R,\end{aligned}$$

where all parameters are greater than 0 and new type of parameter σ is the fraction of newborns that are healthy.

- (a) Interpret each variable and parameter in practical terms.
 - (b) Draw a flowchart of this model.
 - (c) Show that this system has two quantities that are conserved in time and use this information to reduce the system to three differential equations in S_m , S , and I .
 - (d) Determine the disease free equilibrium for this reduced system.
 - (e) Use the Jacobian approach to compute the basic reproduction number.
 - (f) Use the next-generation approach to compute the basic reproduction number.
2. pg. 119, #5.2 part (a) only.
 3. pg. 119, #5.3 part (a) and (b) only.
 4. pg. 199, #5.4 part (a) and (b) only.
 5. pg. 120, #5.5 part (a) and (c) only.