MST 383/683
Homework #7

Due Date: Never

1. Consider the following optimization problem:

$$\min_u \int_1^2 \left(tu(t)^2 + t^2x(t)^2\right) dt$$
$$\text{subject to: } \dot{x} = -u(t) \text{ and } x(1) = 1.$$

   (a) From first principles, derive the necessary conditions for a minimizer.
   (b) Using the Hamiltonian, derive the necessary conditions for a minimizer.
   (c) Solve the necessary conditions to determine a (candidate) minimizer for this problem.

2. Consider the following optimization problem:

$$\min_u \int_0^1 \left(x(t)^2 + x(t) + u(t)^2 + u(t)\right) dt$$
$$\text{subject to: } \dot{x} = u(t) \text{ and } x(0) = 0.$$

   (a) From first principles, derive the necessary conditions for a minimizer.
   (b) Using the Hamiltonian, derive the necessary conditions for a minimizer.
   (c) Solve the necessary conditions to determine a (candidate) minimizer for this problem.

3. Consider the following optimization problem:

$$\min_u \frac{1}{2} \int_0^1 \left[(x(t) - t - 1)^2 + u(t)^2\right] dt$$
$$\text{subject to: } \dot{x} = u(t) \text{ and } x(0) = 1.$$

   (a) From first principles, derive the necessary conditions for a minimizer.
   (b) Using the Hamiltonian, derive the necessary conditions for a minimizer.
   (c) Solve the necessary conditions to determine a (candidate) minimizer for this problem.