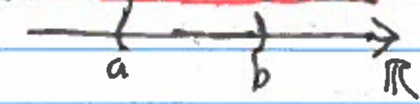
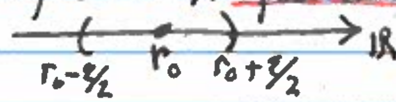
Lecture 6: Planar Sets

Functions of 1-variable sets

- (a,b): open interval

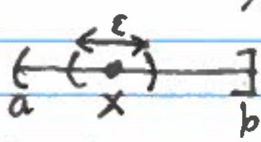


- $(r_0-\varepsilon/2, r_0+\varepsilon/2)$: open ball of radius ε centered at r_0.

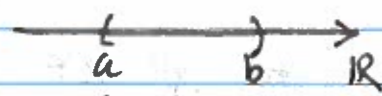


Also called an open disk or circular neighborhood of radius ε about r_0

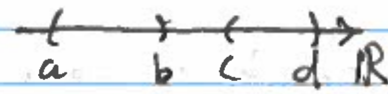
- A point $x \in I$ is an interior point if there exists ε such tat $(x-\varepsilon/2, x+\varepsilon/2) \subset I$



- If for all $x \in I$, x in an interior point of I then I is open.



(a,b) - open



$(a,b) \cup (c,d)$ - open

- If I^c is open then I is closed.

$[a,b]$ - closed

$[a,b]^c = (-\infty, a) \cup (b, \infty)$

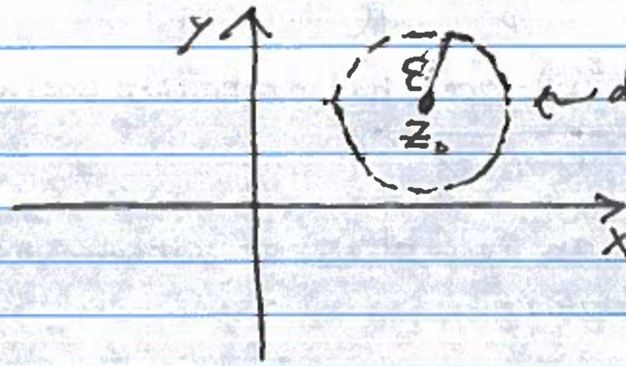
Sets in $\mathbb{C}$:

- Open disk of radius ρ centered at z_0:

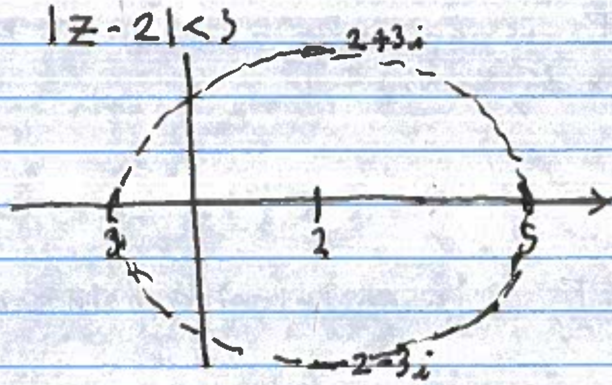
$$|z-z_0|<\rho$$

Short for:

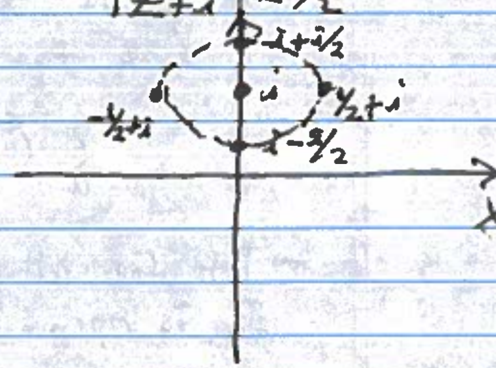
$$\{z\in\mathbb{C} : |z-z_0|<\rho\}.$$



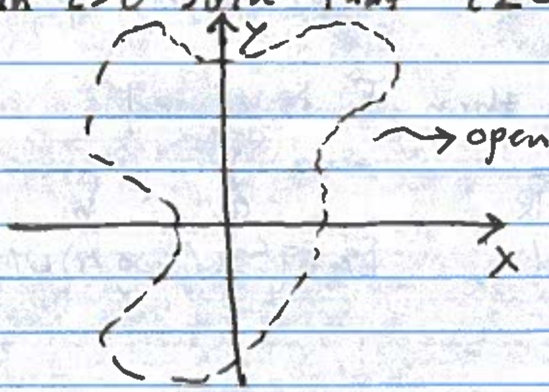
$|z-2|<3$

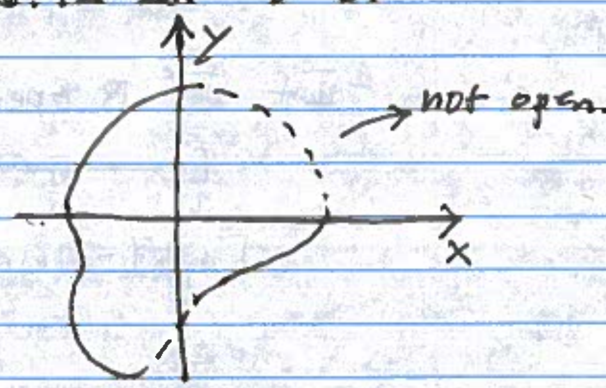


$|z-i|<\frac{1}{2}$

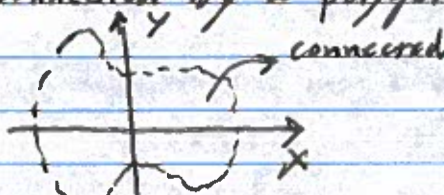


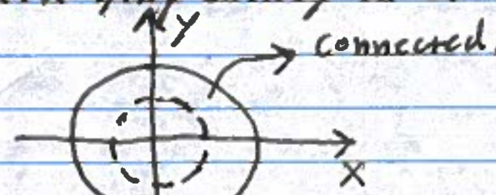
- A set S is open if for all $z_0\in S$ there exists an $\varepsilon>0$ such that $\{z\in\mathbb{C} : |z-z_0|<\varepsilon\}\subset S$.



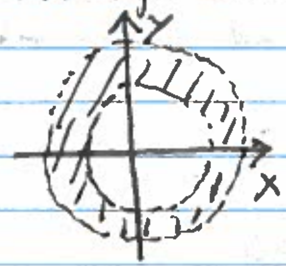


- A set S is connected if any two points $z_1, z_2\in S$ can be connected by a polygonal path lying entirely in S.

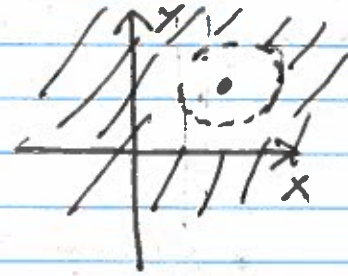




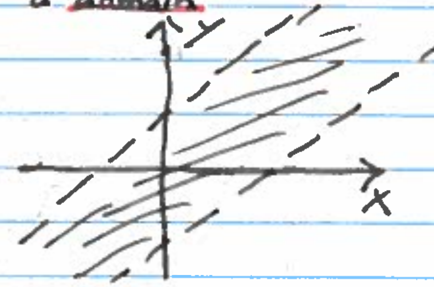
- An open connected set is called a domain



domain



domain



domain

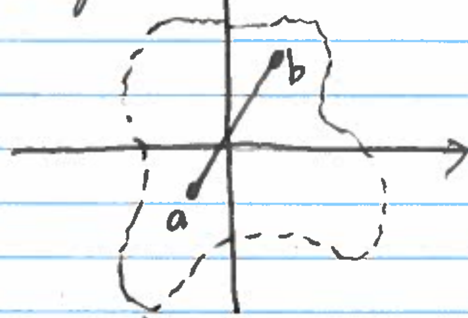
Theorem - Suppose $u(x,y)$ is a real valued function defined on a domain D. If the partial derivatives of u satisfy

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$$

on D, then $u = \text{const.}$ on D.

proof:

Let $\ell(t) = (x(t), y(t)) = (at+b, ct+d)$ be a line segment in D:



Define $F(t)$ by

$$F(t) = u(x(t), y(t))$$

$$\frac{dF}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

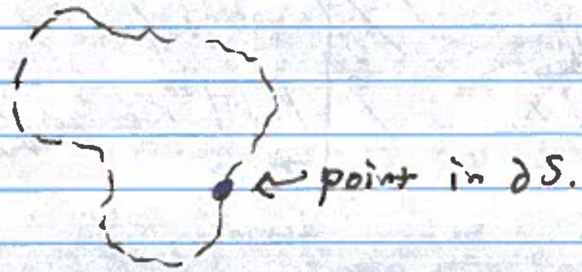
$$= a\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial y}$$

$$= 0$$

$\Rightarrow$ F is constant

$\Rightarrow$ u is constant allong all lines.

- A point $x \in S$ is a boundary point if every neighborhood of x contains a point in S and one not in S.



- ∂S the set of all boundary points is called the boundary of S.