MTH 351/651
Homework #4

Due Date: September 30, 2022

1   Problems for Everyone

1. For each of the following dynamical systems on the circle S^1, find and classify all the fixed points, and sketch the phase portrait on the circle.

   (a) $\dot{\theta} = 1 + 2\cos(\theta)$

   (b) $\dot{\theta} = \sin^3(\theta)$

   (c) $\dot{\theta} = 3 + \cos(2\theta)$

   (d) $\dot{\theta} = \sin(\theta) + \cos(\theta)$

2. At 12:00, the hour and minute hands of a clock are perfectly aligned. When is the next time they will be aligned?

3. For each of the following dynamical systems on the circle S^1, sketch all of the qualitatively different phase portraits on S^1 that occur as μ is varied. Classify the bifurcations that occur as μ varies and find all the bifurcation values μ. **Hint:** Do not forget that in addition to a zero, the sign of a function can change across a vertical asymptote.

   (a) $\dot{\theta} = \mu + \cos(\theta) + \cos(2\theta)$,

   (b) $\dot{\theta} = \dfrac{\sin(\theta)}{\mu + \sin(\theta)}$.

   (c) $\dot{\theta} = \dfrac{\sin(2\theta)}{1 + \mu\sin(\theta)}$.