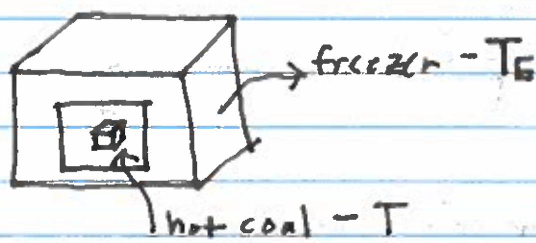
Lecture 1: Intro Example

Classic Physics - Derive a system of differential equations and solve them. The entire solution of the system is determined from initial conditions.

Example - An object is placed in a cold (hot) environment at a fixed temperature T_E



What is the temperature of the object as a function of time?

Mathematical Model:

1. $\frac{dT}{dt} = f(T - T_E)$, $T(0) = T_0$

- $f(0) = 0$
- $f < 0$ if $T - T_E > 0 \Rightarrow T > T_E$
- $f > 0$ if $T - T_E < 0 \Rightarrow T < T_E$

Newton's Law of Cooling:

$$f(T - T_E) = -K(T - T_E)$$ *simplest function which satisfies properties*

$$\frac{dT}{dt} = K(T - T_E)$$

↳ units of inverse time.

Solution:

Let $\Delta T = T - T_E$

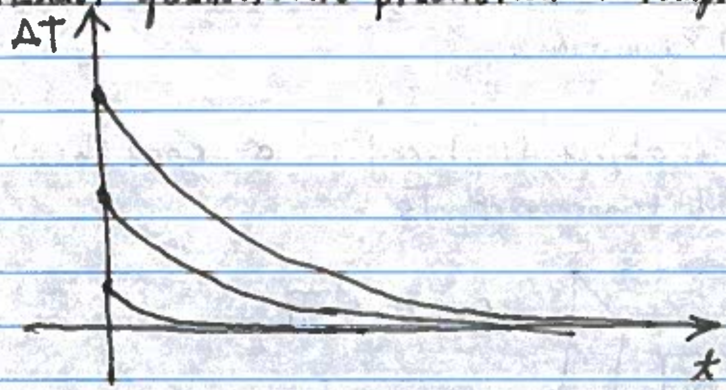
$$\Rightarrow \frac{d\Delta T}{dt} = \frac{dT}{dt} = -K(T - T_E) = -K\Delta T$$

$$\Rightarrow \int_{\Delta T_0}^{\Delta T} \frac{1}{\Delta T} \, d\Delta T = \int_0^t -K \, dt$$

$$\Rightarrow \ln\left(\frac{\Delta T}{\Delta T_0}\right) = -Kt$$

$$\Rightarrow \Delta T = \Delta T_0 e^{-Kt}$$

*Exact quantitative prediction of temperature



2. Nonlinear Newton's Law of Cooling

$$\frac{d\Delta T}{dt} = -K\Delta T - \alpha \Delta T^3$$

$$\Delta T(0) = \Delta T_0$$

Note: If we think of $\frac{d\Delta T}{dt} = f(\Delta T)$ then this is the first two terms in a Taylor series expansion.

$$\Rightarrow -t = \int_{\Delta T_0}^{\Delta T} \frac{1}{Ks + \alpha s^3} ds$$

$$= \int_{\Delta T_0}^{\Delta T} \left(\frac{1}{Ks} + \frac{K-\alpha}{\alpha} \frac{1}{K + \alpha s^2} \right) ds$$

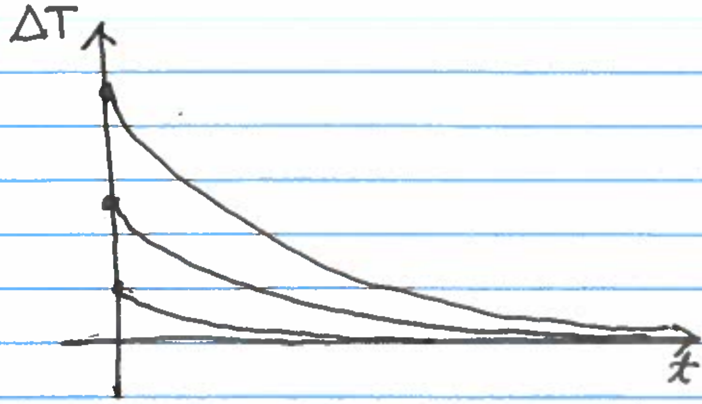
$$= \frac{1}{K} \ln\left(\frac{\Delta T}{\Delta T_0}\right) + \frac{K-\alpha}{K\alpha}\sqrt{\frac{K}{\alpha}}\left(\tan^{-1}\left(\sqrt{\frac{K}{\alpha}}\Delta T\right) - \tan^{-1}\left(\sqrt{\frac{K}{\alpha}}\Delta T_0\right)\right)$$

To get a complete solution need to solve for ΔT...

*Different approach:

- $\frac{d\Delta T}{dt} = 0$ when $\Delta T = 0$
- $\frac{d\Delta T}{dt} < 0$ when $\Delta T > 0$

$\Rightarrow \Delta T$ is monotone decreasing, flattens out to 0.



All initial conditions go to 0; not that qualitatively different than using the linear approximation.