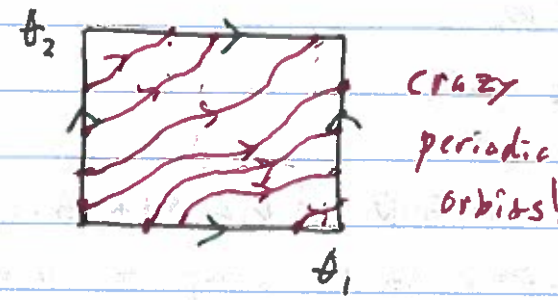
Lecture 22: Motion on Surfaces

Motion on a torus:

$\dot{\theta}_1 = \omega_1 + K_1 \sin(\theta_2 - \theta_1)$

$\dot{\theta}_2 = \omega_2 + K_2 \sin(\theta_1 - \theta_2)$



Let $\phi = \theta_2 - \theta_1$

$\dot{\phi} = \omega - K\sin(\phi)$

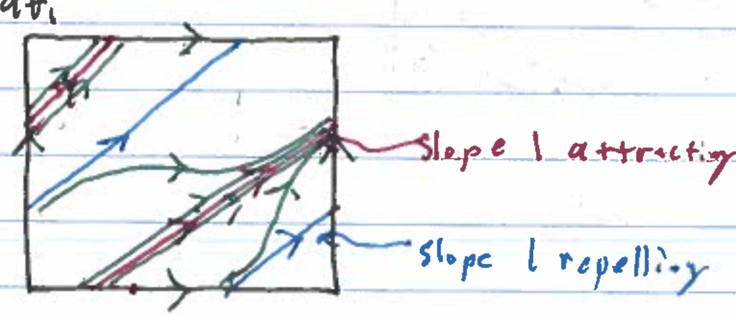
$\omega = \omega_1 - \omega_2, \; K = K_1 + K_2$

If $|\omega/K| < 1$ two fixed points where $\sin(\phi^*) = \omega/K$

$\Rightarrow \dot{\theta}_1 = \omega_1 - K_1\left(\dfrac{\omega_1 - \omega_2}{K_1 + K_2}\right) = \dfrac{K_2\omega_1 + K_1\omega_2}{K_1 + K_2}$

$\dot{\theta}_2 = \dfrac{K_2\omega_1 + K_1\omega_2}{K_1 + K_2}$

$\Rightarrow \dfrac{d\theta_2}{d\theta_1} = 1$ at equilibrium



Uncoupled Systems

$\dot{\theta}_1 = \omega_1$

$\dot{\theta}_2 = \omega_2$

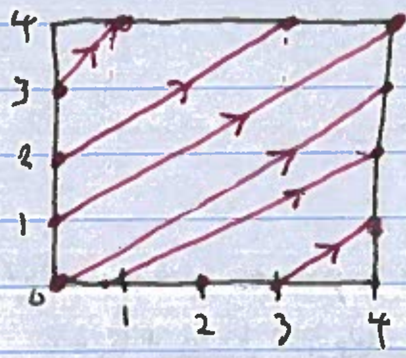
$$\frac{d\theta_2}{d\theta_1} = \frac{\omega_2}{\omega_1}$$

If $\omega_2/\omega_1 \in \mathbb{Q}$, $\exists\, p,q$ such that $\omega_2/\omega_1 = p/q$.

$\Rightarrow$ when θ_2 completes p revolutions, θ_1 completes q revolutions.

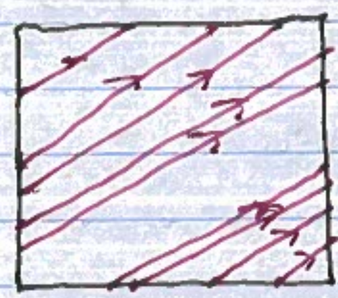
Example:

$$\frac{d\theta_2}{d\theta_1} = \frac{3}{4}$$



Example:

$$\frac{d\theta_2}{d\theta_1} = \sqrt{2}$$



quasiperiodicity $\rightarrow$ trajectories never close.