MTH 381
Homework #1

1 Theory Problems

1. pg. 12, #3
2. pg. 7, #3, #4, #5,#6, #11, #12, #14

2 Applied Problems

1. pg. 7, #2
2. pg. 12, #5
3. Enumerate the rationals in $[0,1]$ as $r_1, r_2, \ldots$ and define

$$D_n(x) = \begin{cases} 1, & \text{if } x \in \{r_1, r_2, \ldots, r_n\} \\ 0, & \text{otherwise} \end{cases}.$$

   (a) Show directly that D_n is Riemann integrable.

   (b) Prove that $D_n(x)$ converges pointwise to the Dirichlet function $D(x)$.

   (c) Show that $D_n(x)$ does not converge uniformly to $D(x)$.

4. Consider the sequence of functions defined by

$$D_n(x) = \cos(n!\pi x)^{2n}.$$

   (a) Using some kind of software, plot $D_1(x), \ldots, D_5(x)$. No need to print this out or anything. I just want you to see these plots for your own benefit.

   (b) Since each D_n is continuous it is Riemann integrable. Compute

$$\int_0^1 D_n(x)dx.$$

   You can use your Calc. 2 knowledge to do this problem.

   (c) Prove that $D_n(x)$ converges pointwise to the Dirichlet function $D(x)$.

   (d) Prove that $D_n(x)$ does not converge uniformly to $D(x)$.