# MTH 381
# Homework #7

Due Date: November 11, 2022

## 1 Theory Problems

1. Nah

## 2 Applied Problems

1. pg. 99, #1, #2, #5. pg. 100, #12.
2. Compute
$$\lim_{n\to\infty} \int_0^1 x^n \sin(x) dx.$$
3. Let $f : \mathbb{R} \mapsto \mathbb{R}$ be integrable meaning
$$\int_{-\infty}^{\infty} |f(x)| dx < \infty.$$
The Fourier transform of $f$, denoted by $\hat{f}$, is defined by
$$\hat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx.$$
This integral should be interpreted as the sum of two real valued integrals of the real and imaginary parts.
   (a) Prove that $\hat{f}(k)$ is bounded.
   (b) Prove that $\hat{f}(k)$ is a continuous function. **Hint:** Let $h_n \to 0$ and use the dominated convergence theorem to show that
$$\lim_{n\to\infty} \left( \hat{f}(k + h_n) - \hat{f}(k) \right) = 0.$$
4. Compute the following limit:
$$\lim_{n\to\infty} \int_0^n \left(1 + \frac{x}{n}\right)^{-n} dx.$$
5. Compute the following limit
$$\lim_{n\to\infty} \int_0^n \left(1 + \frac{x}{n}\right)^{-n} x^{-1/n} dx.$$

6. Give an example of functions $\phi_n$ on $[0,1]$ for which

$$\lim_{n\to\infty} \int_0^1 \phi_n(x)dx = 0$$

while

$$\lim_{n\to\infty} \phi_n(x)$$

exists for no $x \in [0,1]$.