Lecture 2: Why Care?

1. If $f_n:[0,1]\to\mathbb{R}$ are continuous, have finite lengths, and converge uniformly to f, does f have finite length?

Connect the points

$$(1,0),\left(\frac{2}{3},\frac{-2}{3}\right),\left(\frac{2}{5},\frac{2}{5}\right),\dots,\left(\frac{2}{2n+1},\frac{(-1)^n 2}{2n+1}\right),(0,0)$$

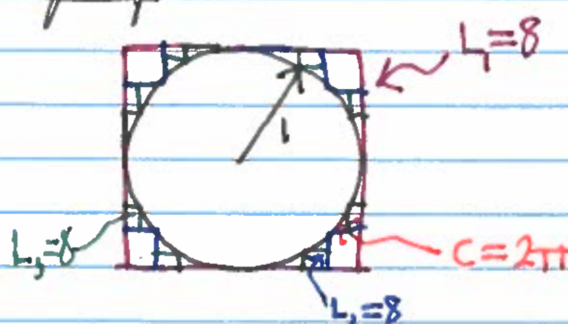
The length of each segment is greater than $2/3, 2/5, \dots$

The length of the limiting curve is given by

$$\lim_{n\to\infty}\sum_{n=1}^{\infty}\frac{2}{2n+1}=\infty.$$

2. $\pi=4$

~~proof~~



$$\lim_{n\to\infty}L_n=\lim_{n\to\infty}8=8=2\pi$$

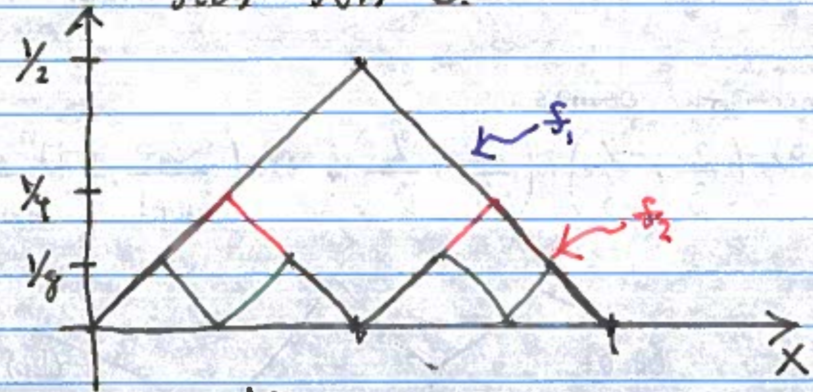
$$\Rightarrow \pi=4.$$

This example shows the limitations of not having "rigorous intuition."

3. Minimize the following functional

$$I[f] = \int_0^1 f(x)^2\,dx + \int_0^1 (f'(x)^2 - 1)^2\,dx$$

$$f(0) = f(1) = 0.$$



$I[f_n] \leq 1/2n \to 0.$

$f_n \to 0$

However, $I[0] = 1$??

What does it mean $f_n \to 0$, what about f_n' what does the sequence of derivatives converge to??